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Stochastic Programming Model in Least Cost Feed Formulation for Lactating Cattle

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Abstract. A conventional linear programming model (LPM) for feed formulation of lactating cattle will overlook the variation in feed components. LPM only considers the mean composition of feed values, regardless of variations, the confidence in satisfying the nutrient need falls to 50%. Whereas the stochastic model (SM), which takes into account both the mean and variation of feed composition and provides 90-99% confidence in meeting the nutrient need. In present work, we have proposed SM for least-cost feed formulation of lactating cattle where the variation in the composition of nutrients like crude protein (CP), Calcium (Ca) and Phosphorus (P) in the feedstuff are considered. Data provided by the National Research Council (2001) are the basis for the current analysis. These SMs are resolved using M.S. Excel's Generalized Reduced Gradient (GRG) nonlinear and LINGO's Nonlinear solver, and the results are compared to LPM; the feed formulated by SM (90 % and 99 %) has the lowest cost when compared to LPM. Nutrients estimated by LPM, SM by GRG nonlinear, and SM by Nonlinear solver utilized for feed formulation had no significant differences as ($p>0.05$). When compared to LPM, the stochastic model is a better technique, particularly when dealing with nutrient variation.

Keywords: feed formulation, Generalized Reduced Gradient, Linear Programming Model, LINGO's Nonlinear Solver, stochastic model

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1. Introduction

Ration formulation is a procedure that combines several feed components to provide animals with the nutrients they require for a predetermined amount of time. A ration should contain all of the vital nutrients and energy that animals require to maintain their vital physiological processes of growth, reproduction, and health. The ideal ratio should be affordable, environmentally sustainable, palatable, digestible, and include a moderate amount of antinutritive components.

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According to research, the amount of nutrients including minerals, glucose, amino acids, fatty acids, and other substrates a farm animal has access to affects how quickly they grow and how much milk they produce [1] – [5]. Animal nutrition also varies depending on the animal's life cycle and activity, such as serving as a protein supply, a seed stock, or a breeder. A diet with an imbalanced nutrient profile can lead to malnutrition, illnesses, and performance issues. To avoid these physiological problems, the animals must be fed specific nutrients[6]–[8]. For instance, for beef cattle, the nutrients DMI, TDN, CP, Ca, and P are prioritised, but for dairy cattle, the nutrients Met, Lys, Arg, Thr, Leu, Ile, Val, His, Phe, Trp, ME, Ca, and Pare are prioritised [9]. The amount of nutrients needed is also influenced by the local feed supply. The formulation of the feed or the optimization of the feed to minimise cost concerns while adding multiple feeds in appropriate quantities to meet the nutrient requirements.

In actual circumstances, the content of nutrients might significantly alter from one batch of ingredients to another batch because they may come from various sources [10]. Numerous researchers have studied about generating cow feed using linear programming, and the majority of commercial feed formulation software follows this strategy [11]. This method considers the average value of these analytical parameters to be disregarding variability in nutrients when producing feed formulation. The ideal feed formulation for cattle has been the subject of research in recent decades [12]. The feed formulation or the optimizing the feed at least cost concerns with adding several feeds with sufficient quantity such that it satisfies the nutrient requirement. Forages like as silage and hay, as well as concentrates such as rolled grain, meals, hulls, seeds, and minerals, were utilised in the formulation [13]. Lactating cattle require a proper ration for milk production (with a minimum of 4% fat), body maintenance, during the third trimester of pregnancy. As a result, producers face a tremendous difficulty in developing the most cost-effective ration with the essential nutrients for these cattle's foetal growth [14].

In real scenarios, the composition of nutrients can vary considerably from one batch of ingredients to another batch as it may come from different sources. Many researchers have considered the formulation of cattle feed through linear programming and most of the commercial feed formulation software uses a linear programming approach. In this approach, the average value of these analytical factors are considered to neglect variability in nutrients while formulating feed formulation [15]. The stochastic model (SM) can be used to reduce variance in nutrient values and increase the probability of satisfying nutrient requirements when compared to the linear programming model (LPM) [16]. Van De Panne constructed a stochastic model for the lowest cost ration, with variable protein as probabilistic constraints and fat as linear constraints, and compared the results to LPM, which ignores feeds with higher variances in protein nutrient value [17]. This leads to the possibility of long-term nutritional contentment. For least-cost poultry rations, linear programming with stochastic constraints (nonlinear) is reduced to linear constraints using the approximation method [18]. The stochastic programming model for the least-cost feed formulation can be addressed using an iterative quadratic programming method under a probabilistic constraint as well as other linear constraints [19] Pesti 1999 described how to solve stochastic programming for feed formulation using an electronic spreadsheet. Least-Cost Ration Formulations for Holstein Dairy Heifers were designed using stochastic programming, and it was found that altering the nutrient content through a safety margin or right-hand side adjustment reduced the cost of the ration and the level of CP overfeeding [20].

In 2011, stochastic programming was developed in Nigeria's Semi-intensive Culture System for African Catfish (Clarias gariepinus) Least-cost Feed Formulation at 50, 60, 70, 80, and 85 percent probability ranges for African Catfish (Clarias gariepinus) Least-cost Feed Formulation. The results showed that as the formula's probability level increased from 60 to 85 percent, thereby eliminating the issue of nutritional variance [8]. Hence the same method is applied to get the least cost feed formulation for lactating cattle. The challenge in constructing the feed (ration) is that lactating cattle are often fed a ration consisting of forages such as silage, hay, and concentrates, all of which have varying nutrient content [21]. This has a negative impact on the rate of animal development. Lactating cattle needs a sufficient supply of energy and protein through the lactating cycle. The time between one calving and the next is known as the lactation cycle. The lactation cycle is classified into three phases: early lactation, which lasts 14-100 days, middle lactation, which lasts 100-200 days, and late lactation, which lasts 200-305 days [22]. Cattle calve every twelve months in an ideal environment, because they must calve in order to produce milk. If the ration lacks the necessary nutrients, metabolic diseases such as fatty liver, ketosis owing to energy deficiency, and low calcium milk etc [23]. When no fluctuation in the nutrient is seen, the LPM can be employed efficiently. However, in countries like India with diverse geographic regions and drastically varying climatic conditions, feed composition and nutrient availability also alter, impacting the animal's growth. Even when the range of nutrients is negligible, it may not have an immediate impact on the animal, but over the course of time, it will have a significant impact on the animal's growth at various stages. Taking into account all of these factors, the current study's purpose is to identify the necessary nutrients for nursing cattle and to comprehend nutrient variability in feeds such as CP, Ca, and P. Using a stochastic model to create a low-cost ration for lactating cows. The results of the stochastic models are compared to the LPM using M.S. Excel and the LINGO application.

2. Materials and Methods

The current analysis is based on data gathered by the National Research Council (2001). Since the early 20th century, the National Research Council (NRC) has published nutrient requirement guidelines for the majority of economically significant farm animal species. Significant changes were made to the Nutrient Requirements of Dairy Cattle in the NRC's seventh updated edition (1989), which was published in 2001. A computer model developed by the 2001 Dairy NRC incorporates calculations of nutrient requirements and their interactions, enabling estimates of nutrient requirements and dynamic ration assessment.

Table 1 shows the compositions of feedstuffs with nutritional levels of CP, Ca, and P with variation. LPM and stochastic models were introduced at two different months of pregnancy with variable body weights. For the 6th and 7th months of pregnancy, the required nutrition for breastfeeding cattle is discussed below and shown in Table 2. Specific estimates for nutrient accretion rates in body concepts are needed to predict nutritional requirements throughout late pregnancy [24]. The feedstuff with variable names is presented below same has been used in all Tables.

X1	Corn silage, matured	X11	Corn grain, ground dry	X19	Soya bean hulls
X2	Sorghum grain Silage	X12	Sorghum grain, dry rolled	X20	Calcite (Calcium Carbonate) CaCo3
X3	Grass silage	X13	Wheat grain rolled	X21	DCP (Di calcium phosphate) CaHPO4
X4	Legume silage	X14	Barley grain rolled	X22	Lime stone, ground
X5	Grass-Legume mixed silage	X15	Corn gluten meal	X23	Salt (Sodium Chloride) NaCl
X6	Sorghum Sudan hay	X16	Cottonseed meal		
X7	Grass hay	X17	Alfalfa Meal		
X8	Legume hay	X18	Cotton seed hulls		

Table 1. Composition of Feedstuffs (DM, CP, TDN, Ca, P) Commonly Fed to Dairy Cattle (all values on a dry basis)* collected from NRC (2001)

Feeds	Type	Cost (/kg)	DM (%)	CP (%)	TDN (%)	Ca (%)	P (%)	Max (% D.M.)
X1	F	5	44.2	8.5 ±.3.9	65.4	0.26 ±0.1	0.25 ±0.04	30
X2	F	5	28.8	9.1 ±2.6	56.7	0.5 ±0.26	0.21 ±0.08	30
X3	F	5.5	36.5	12.8 ±3.7	55.7	0.55 ±0.08	0.29 ±0.08	30
X4	F	5	39.1	20 ±3	56.6	1.34 ±0.26	0.32 ±0.06	30
X5	F	5	38.5	15.4 ±2.4	53.6	1.06 ±0.27	0.33 ±0.05	30
X6	F	16.5	86.5	9.4 ±2.2	54.4	0.54 ±0.21	0.2 ±0.06	20
X7	F	18	88.1	10.6 ±3.1	56.3	0.58 ±0.23	0.23 ±0.06	20
X8	F	17	87.8	20.2 ±2.6	58.9	1.52 ±0.27	0.26 ±0.05	20
X9	F	18	84.7	13.3 ±3.3	57	0.97 ±0.17	0.37 ±0.08	20
X10	F	5	92.7	4.8 ±1.9	47.5	0.31 ±0.22	0.1 ±0.05	20
X11	C	11.5	88.1	9.4 ±1.3	88.7	0.04 ±0.07	0.3 ±0.05	10
X12	C	30	88.6	11.6 ±1.8	80.6	0.07 ±0.04	0.35 ±0.07	10
X13	C	45	89.4	14.2 ±2.3	86.6	0.05 ±0.03	0.43 ±0.14	10
X14	C	18	91	12.4 ±2.1	82.7	0.06 ±0.02	0.39 ±0.06	10
X15	C	35	86.4	65 ±7.8	84.4	0.06 ±0.04	0.6 ±0.28	10
X16	C	30	90.5	44.9 ±4.1	66.4	0.2 ±0.1	1.15 ±0.1	10
X17	C	30	90.3	19.2 ±3.3	56.4	1.47 ±0.36	0.28 ±0.07	10
X18	C	12	89	6.2 ±3.6	34.3	0.18 ±0.1	0.12 ±0.06	10
X19	C	14.5	90.9	13.9 ±4.6	67.3	0.63 ±0.07	0.17 ±0.07	10
X20	M	1.7	100	0	0	30.71	0	0.5
X21	M	37	97	0	0	22	0	2
X22	M	3.5	100	0	0	34	0	2
X23	M	4.1	100	0	0	0	0	0.5

*F = forage, C = concentrates, M = Minerals.
The cost of feeds is taken from (https://www.indiamart.com/). Maximum limit set for feeds is 20% in kept in 30%, hay is 20%, and concentrates are 10% of DMI. Mineral limits are set accordingly.

Table 2. Minimum Nutrient (DMI, CO, TDN, Ca & P) Requirements of Two Cattle that Have Been Taken for Studies*

Cattle	**Models**	**Minimum Nutrient Requirements**				
		DMI (kg)	**CP (g)**	**TDN (g)**	**Ca (g)**	**P (g)**
Cattle-1	BW:400 kg MY: 15kg M.F.: 4.5% Months of Pregnancy:7	15.647	2091.586	9363.730	180.381	32.545
Cattle-2	BW:380 kg MY: 14kg M.F.: 4% Months of Pregnancy:6	14.389	1932.147	8490.461	84.460	29.161

*Nutrients are calculated using the above-discussed equations

2.1. Nutritional Analysis of Dairy Cattle

There is an immense potential for dairy cattle to yield carbohydrates, proteins and fats, but they still have a very strong need for nutrients. The dairy cattle require 4 % of DMI of their Bodyweight to gain maximum body gain [25]. The provisions of nutrients for lactating cattle are DMI (kg/d), CP, TDN; Ca & P (g/d) are evaluated by using the NRC-2001 & ICAR-2013 standards are discussed below. In addition to the 4 to 5 L for each liter of milk obtained, lactating dairy cattle need 60 to 70 L of water per day for maintenance.

2.1.1. Dry Matter Intake (DMI) (kg/d)

The DMI is profoundly important because it determines the amount of nutrients that are available for the health and development of an animal. In order to avoid underfeeding or overfeeding of nutrients and to facilitate effective nutrient usage, accurately calculated DMI is essential for the formulation of diets. The DMI equation for lactating cattle's given by NRC 2001 is:

$$\mathrm{DMI}\,(\mathrm{kg/d}) = (0.372 \times \mathrm{FCM} + 0.0968 \times \mathrm{BW}^{0.75}) \times (1 - e^{(-0.192 \times (\mathrm{WOL} + 3.67))}) \tag{1}$$

Where FCM = 4 % fat corrected milk (kg/day), BW = body weight (kg), and WOL = week of lactation. The term $(1- e^{(-0.192\times(\mathrm{WOL} +3.67))})$ adjusts for depressed DMI during early lactation. The equation (1) was compared with those developed by [9] and actual DMI, which shows that it is lightly under-predicted in the mid-lactation. Rendering to the ICAR-2013, the DMI requirement depending on body weight , milk production & pregnancy is formulated as follows:

$$\mathrm{DMI}\,(\mathrm{kg/d}) = \mathrm{BW}^{0.75} \times 0.0968 + (0.372 \times (\mathrm{MY}) \times (0.4 + 0.15 \times \mathrm{MF})) + (0.14 \times \mathrm{P} + 0.01) \tag{2}$$

Where BW = Body weight, MY = Milk yield, M.F. = Milk fat, and P = Pregnancy month. The DMI calculated by equation (2) gradually increases with days of lactation, whereas DMI by equation (1) remains unchanged after middle lactation. The mean deviation & standard error of DMI calculated by NRC & ICAR-2013 is 0.64±0.24. Therefore in the present study, the equation (2) is preferred to calculate the DMI (kg/d) for lactating cattle.

Climatic conditions outside the thermal neutral (5 to 20°C) zone influence the DMI of lactating cows [26], discussed about the changes in DMI if temperatures are outside the thermal neutral region. The subsequent changes happen in DMI:

- temperatures higher than 20° C- $\mathrm{DMI} \times (1\text{-}\ ((°\ \mathrm{C}\text{-}20)\ \times 0.005922))$
- temperatures less than 5° C, $\mathrm{DMI}/(1\text{-}((5°\ \mathrm{C}) \times\ 0.004644))$.

However these are adjustment factors to be included while calculating DMI for cattle. Due to inadequate DMI data outside the thermal neutral region, we did not include a temperature or humidity adjustment factor in the equation used to predict lactating cow DMI .

2.1.2. Crude Protein (CP) & Total Digestible Nutrient (TDN) (g/d)

Dietary protein generally refers to a crude protein (CP). CP is essential for maintenance, reproduction, growth, and lactation of dairy cattle. Protein over lactation requirements has been shown to have negative effects on reproduction. Dietary protein levels below 6-8% in the ration may decrease the DMI of cattle. This often occurs when these animals are mainly fed on straws, stover, and other low-quality roughages and in this cases, the supplementary protein will support the DMI [27].

Several cases have reported that feeding diets containing 19 % or more CP in diet DMI lowered conception rates. Total Digestible Nutrient (TDN) has been in use for long to indicate the energy content of a feed as well as the energy requirement of animals. Cattle need energy for maintenance, activity, pregnancy, milk production and for gaining body condition. The equation for CP (g//d) & TDN (g/d) are calculated based on the requirement tables given in ICAR-2013, discussed in detail in [28].

2.1.3. Calcium (Ca) & Phosphorus (P) (g/d)

The micro minerals like Ca & P are essential for growth & reproduction of cattle. Macro minerals are essential significant elements of bone and other structures and are key parts of body fluids. The ratio and percentages of Ca and P are important considerations in formulating rations for dairy cows. The minimum requirement of Ca for the reproduction in dairy cattle was determined to be in the range of 0.16 to 0.18% of the total ration on a dry matter basis. Severe deficiencies of P in cattle reduce feed intakes, feed efficiencies, and retard the growth [29] The requirement for P, as a per cent of dry matter in rations for lactating cows, is given by the National Research Council (NRC-1978) as 0.40% for high producing and 0.31% for low producing cows. Production of milk was lowered more from cows fed rations containing 0.3% phosphorus than from cows fed rations containing 0.55% phosphorus. The equations to Calculating Ca (g/d) & P (g/d) for milking cattle is taken from NRC (2001) [30]. Nutrient requirements for 2 different months of pregnancy with different body weights, Milk yield & Milk fat are given in Table 2.

2.2. Least Cost Feed Formulation Models

Linear programming model has long been a standard in the formulation of least-cost rations [31]. The assumption of deterministic parameters which assume that all coefficients are known with certainty in the model. This assumption does not take place due to the inconsistency in nutrient values in the feed formulation problem. Nutrient values are available as the real average composition estimates [32]. A linear model to minimize feed costs, with 23 decision variable, one equality constraint, six inequality constraints and a feed limit is given below:

Objective Function: $$\text{Min } Z = \sum_{j=1}^{23} c_j x_j \quad (3)$$

Subject to Constraints:

DMI: $$\sum_{j=1}^{23} a_{ij} x_j = b_i \,;\, i = 1 \quad (4)$$

TDN: $$b_{i\min} \le \sum_{j=1}^{23} a_{ij} x_j \le b_{i\max} \,;\, i = 3 \quad (5)$$

CP, Ca, P: $$b_{i\min} \le \sum_{j=1}^{23} a_{ij} x_j \le b_{i\max} \,;\, i = 2,4,5 \quad (6)$$

Roughages: $$b_{i\min} \le \sum_{j=1}^{10} x_j \le b_{i\max} \,;\, i = 6 \quad (7)$$

Concentrate: $$b_{i\min} \le \sum_{j=11}^{23} x_j \le b_{i\max} \,;\, i = 7 \quad (8)$$

Feeds: $$0 \le \sum_{j=1}^{23} x_j \le b_{i\max} \,;\, i = 8 \quad (9)$$

Where c_j = cost per unit for the jth feedstuff, b_i = nutritional requirements for nutrient i, a_{ij} = quantity of the ith nutrient per unit of the jth feedstuff, and x_j = quantity of the jth feedstuff. As a_{ij} in case of C.P., Ca & P are mean nutrient values of the composition of feeds. LP ensure only 50 % of chance of satisfying them. In order to increase the chance of satisfying these nutrients to 90-99%, must apply the certain probabilities.

Stochastic Model with Nonlinear Constraints, the nutrient feedstuff values vary even in the same lot. Since in practice, it's not really possible to take samples from each quantity of the inputs, the coefficient denoting the nutrient values are not fully known. As a rule, the mean values of these coefficients are known, but when these values which are used to find optimized feed formulation using LP, some quantities of cattle feed will not meet the requirement. This is crucial when the minimum values of some nutrients are guaranteed, like in the case of protein, Calcium and Phosphorus content in lactating cattle feeds. If this content of any feeds varies, then, while using the mean values for optimal composition, one can never be sure that feed formulation satisfies the nutrient requirement.

This difficulty can be dealt with changing the minimum requirement of CP, Ca and P with probabilistic one. By implementing the probability of CP, Ca and P content in the feed formulation equal to or larger than an absolute minimum should not be lower than given levels (say, 0.90, 0.95, 0.99). Then the constraints in the equation (2.4) becomes

$$P\left(\sum_{j=1}^{23} a_{ij}x_j \geq b_{imin}\right) \geq p_\alpha \tag{10}$$

$$P\left(\sum_{j=1}^{23} a_{ij}x_j \leq b_{imax}\right) \geq p_\alpha \tag{11}$$

Where P(f(x) defines the probability of occurring of event should be higher than the required probability p_α.The limitations of the form (10) can be minimised, given that random variables aij with respective means μ_{ij} and standard deviations σ_{ij} are independent and normally distributed. An equation (10) and (11) can be converted to:

$$\sum_{j=1}^{23} \mu_{ij}x_j + \varphi_i\sqrt{\sum_{j=1}^{23} \sigma_{ij}^2 x_j^2} \geq b_{imin} \tag{12}$$

$$\sum_{j=1}^{23} \mu_{ij}x_j + \varphi_i\sqrt{\sum_{j=1}^{23} \sigma_{ij}^2 x_j^2} \leq b_{imax} \tag{13}$$

The term φ_i is the standard normal deviate corresponding to p_α. if the desired probability of success is $P > 0.95$ in constraint (3.5), then the standard normal deviate is -1.645, because 95% of the standard normal distribution is greater than or equal to -1.645. As the probability of exceeding a minimum requirement increases, the standard normal deviate becomes more negative. Similarly, if the desired probability of success is $P > 0.95$ in Constraint (3.6), then the standard normal deviate is $+1.645$, because 95% of the standard normal distribution is less than or equal to $+1.645$. As the probability of falling below a maximum requirement increases, the standard normal deviate becomes more positive. After applying the probability $P > 0.90, 0.95, 0.99$ to these constraints, it becomes nonlinear. If we replace the linear constraint in (3.4) in LP by Nonlinear constraints (3.7 and 3.8) then we get a stochastic model with the linear objective function, having one equality constraint, three inequality constraints, three nonlinear inequality constraints and feed constraint, given below.

Objective Function: $\text{Min } Z = \sum_{j=1}^{23} c_j x_j$ (14)

Constraints:

DMI: $\sum_{j=1}^{23} a_{ij}x_j = b_i \text{ ; } i = 1$ (15)

TDN: $b_{imin} \leq \sum_{j=1}^{23} a_{ij}x_j \leq b_{imax} \text{ ; } i = 3$ (16)

CP, Ca, P: $b_{imin} \leq \sum_{j=1}^{23} a_{ij}x_j + (\varphi *_i)\left(\sum_{j=1}^{23} \sigma_{ij}^2 x_j^2\right)^{0.5} \leq b_{imax} \text{ ; } i = 2,4,5$ (17)

Roughages: $b_{imin} \leq \sum_{j=1}^{10} x_j \leq b_{imax} \text{ ; } i = 6$ (18)

Concentrate: $b_{imin} \leq \sum_{j=11}^{23} x_j \leq b_{imax} \text{ ; } i = 7$ (19)

Feeds: $0 \leq \sum_{j=1}^{23} x_j \leq b_{imax} \text{ ; } i = 8$ (20)

*Where φ_i is standard normal deviate and the values of these are ± 1.28, ± 1.645, ± 2.33 for 90%, 95% and 99% confidence level.

3. Results and Discussion

Simplex L.P. solver of M.S. Excel solves the LP model and Stochastic models for (90, 95, and 99%) are solved using two different techniques; one is GRG Nonlinear solver of M.S. Excel & nonlinear solver of LINGO application. The GRG algorithm selects a starting value from its calculation in MS excels 2010 and thus leads to different responses on each run as it sets different starting points each time. If all functions and constraints are convex, the GRG approach provides a globally optimal solution. GRG Non-linear is an established reliable approach to solve the nonlinear problem, but it can also solve linear programming issues.

LINGO is a comprehensive tool designed to make it quicker, simpler and more effective to construct and solve Linear, Nonlinear (convex and nonconvex/Global), Quadratic, Quadratically Constrained, Second Order Cone, Semi-Definite, Stochastic, and Integer optimization models. LINGO offers a fully integrated kit that includes a powerful language to express optimization models, a complete functional environment for problems in construction and editing, and a set of fast built-in solvers (https://www.lindo.com). The results obtained by two different methods for 2 different cattle's are shown in Table 3 and Table 4.

Table 3. Least Cost Feed Formulated by Two Techniques for Cattle 1 for L.P. & SM Models on "As fresh" Basis Solved by MS Excel (LP simplex, GRG Nonlinear, and LINGO)

	MS Excel				**LINGO**		
Var	**Simplex LP**	**GRG Nonlinear**			**Nonlinear Solver**		
	LP	**SM 90%**	**SM 95%**	**SM 99%**	**SM 90%**	**SM 95%**	**S.M. 99%**
X1	0.771	0.993	0.628	0.000	0.959	0.943	0.678
X2	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X3	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X4	1.235	2.373	2.797	3.555	2.157	2.311	2.719
X5	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X6	3.129	2.893	2.834	2.704	3.129	3.004	2.861
X7	1.123	0.000	0.000	0.000	0.014	0.000	0.000
X8	3.129	3.129	3.129	3.129	3.129	3.129	3.129
X9	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X10	3.129	3.129	3.129	3.129	3.129	3.129	3.129
X11	1.565	1.565	1.565	1.565	1.565	1.565	1.565
X12	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X13	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X14	0.984	1.565	1.565	1.516	1.558	1.565	1.565
X15	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X16	0.000	0.104	0.151	0.290	0.000	0.062	0.145
X17	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X18	1.565	1.565	1.565	1.565	1.565	1.565	1.565
X19	1.565	1.565	1.565	1.565	1.565	1.565	1.565
X20	0.078	0.078	0.078	0.078	0.078	0.078	0.078
X21	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X22	0.233	0.261	0.265	0.277	0.245	0.250	0.255
X23	0.078	0.078	0.078	0.078	0.078	0.078	0.078
Total Feed (Kg)	18.584	19.298	19.349	19.451	19.171	19.244	19.332
Total Cost (Rs)	229.157	225.523	226.261	228.110	225.122	225.501	226.364

* The maximum limit set for the constraints is 20% of the minimum requirement. Forage & concentrate ratio is 60% : 40%

All of the calculation information is given in Table 3 and 4. The quantity of each feed (X1-X22) is shown in (Kg) and the total feed quantity is shown as 'Fresh' basis in Kg with total cost and cost per kg of ration. Nutrients which are satisfied with the estimated feed (DMI, CP, TDN, Ca and P) are also shown below the cost.

Here, forages such as silage & hay concentrate such as rolled grain, meals, hulls, seeds & minerals are the feeds in the formulation used. For body growth, milk production, lactating cattle need to have an adequate diet. It is, therefore, a severe hurdle to devise the lowest cost for farmers ration with the necessary nutrition for these cattle for foetal development. Thus, stochastic models were developed for 2 different months of pregnancy with different body weights. Cattle-1 is considered to 7 months pregnant and Cattle-2 is deemed to be 6 Month of Pregnant. The cattle model characteristics and nutrient requirements are discussed in Table 2.

Table 4. Least Cost Feed Formulated by Two Techniques for Cattle 2 for L.P. & SM Models on "As fresh" Basis Solved by MS Excel (LP simplex, GRG Nonlinear and LINGO)

Var	MS Excel				LINGO		
	Simplex LP	GRG Nonlinear			Nonlinear Solver		
	LP	SM 90%	SM 95%	SM 99%	SM 90%	SM 95%	S.M. 99%
X1	0.281	0.000	0.000	0.000	0.247	0.000	0.000
X2	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X3	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X4	0.616	2.235	2.313	2.461	1.928	2.206	2.308
X5	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X6	2.878	2.878	2.878	2.878	2.878	2.878	2.878
X7	1.981	0.643	0.564	0.417	0.703	0.672	0.570
X8	2.878	2.878	2.878	2.878	2.878	2.878	2.878
X9	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X10	2.878	2.878	2.878	2.878	2.878	2.878	2.878
X11	1.439	1.439	1.439	1.439	1.439	1.439	1.439
X12	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X13	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X14	0.608	1.382	1.261	1.033	1.405	1.417	1.255
X15	0.131	0.012	0.016	0.025	0.000	0.000	0.000
X16	0.000	0.064	0.209	0.481	0.018	0.035	0.238
X17	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X18	1.439	1.439	1.439	1.439	1.439	1.439	1.439
X19	1.439	1.439	1.439	1.439	1.439	1.439	1.439
X20	0.000	0.011	0.025	0.050	0.000	0.007	0.016
X21	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X22	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X23	0.072	0.072	0.072	0.072	0.072	0.072	0.072
Total Feed (Kg)	16.639	17.369	17.411	17.490	17.323	17.360	17.408
Total Cost (Rs)	221.455	215.758	217.078	219.579	215.128	215.486	217.314

*The maximum limit set for the constraints is 20% of the minimum requirement. Forage & concentrate ratio is 60% : 40 %

The quantity of each feed content is shown in kg and the total feed quantity is shown as "Fresh" basis in kg with total cost and cost per kg of ration. In the feed formulation, information on the amount of forages and concentrate are also shown at the bottom of Tables 3 and 4. It is observed from the obtained results, the total cost of the ration is comparatively lesser in LINGO nonlinear solver than the other two techniques. For Cattle 1, the obtained 19.171, 19.244, and 19.332 kg of feed costs only Rs. 225.12 (SM 90%), Rs. 225.50 (SM. 95%), and Rs. 226.36 (SM. 99%), respectively. Similarly, for Cattle 2, 17.32, 17.36, and 17.40 kg of feed costs only Rs. 215.12 (SM 90%), Rs. 215.48 (SM 95%), and Rs. 217.31 (SM 99%), respectively. Also, the obtained values are very similar to that of the least cost obtained from GRG nonlinear on considering the same SM parameters. Though the calculated data in LP shows variation when compared with the feed cost for cattle 1, i.e., Rs. 229.15 for 18.58 kg, and for cattle 2, i.e., Rs. 221.46 for 16.639 kg, LP cost are numerically higher. Howe where, there is no significant difference (P > 0.05) between the techniques as One-way ANOVA test at 5% level of significance has been performed for the "Null hypothesis: there is no significant difference between the techniques" (Table 5).

Table 5. The one-way ANOVA test at 5% Level of Significance Null Hypothesis that there is no Significant Difference Between the Techniques*

	LP model	SM GRG (99%)	SM LINGO (99%)
Cattle 1	229.16	228.1	226.36
Cattle 2	221.46	219.5	217.31
Mean±S.E	225.31±3.85	223.8±4.3	221.83±4.52
P value			0.8517

*There is no significant difference between techniques as $(p > 0.05)$

The ICAR 2013 and NRC reports are compared to the nutritional requirement stated in the current study. The results were in line with the reported data with only about ±1.5 kg or g of variance. The DMI requirement for the cattle 1 is met and reported to be 15.64kg, which differs from the reported values of ICAR and NRC only by +1.343 and -1.417kg. With a difference of less than 0.5g, CP, TDN, Ca, and P are primarily the same. Because the required values are majorized for the evaluation, these small variations are seen. The limitations would become inflexible if the upper boundaries were not given, which is impossible in practice. In order to prevent this, the nutritional need is assessed by maintaining a minimal range.

Table 6. The comparison of calculated nutrient requirement using LP, SM GRG (99%) and SM LINGO (99%) with reported nutrient requirement by ICAR and NRC

Cattle-1	Required	LP Simplex	SM GRG (90, 95 & 99%)			SM LINGO (90, 95 & 99%)			ICAR-2013	NRC
			SM 90%	SM 95%	SM 99%	SM 90%	SM 95%	S.M. 99%		
DMI (kg)	15.65	15.65	15.65	15.65	15.65	15.65	15.65	15.65	16.99	14.23
CP (g)	2.09	2.09	2.09	2.09	2.09	2.09	2.07	2.15	2.05	2.84
TDN (g)	10.66	10.66	10.66	11.24	11.24	11.24	11.20	11.24	8.26	-
Ca (g)	0.22	0.22	0.22	0.22	0.22	0.22	0.21	0.22	-	0.10
P (g)	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	-	0.04

The sensitivity analysis for the LP simplex approach indicates that feeds X1, X4, X6, X7, X8, X10, X11, X14, X18, X19, X20, X22, and X23 contribute to optimal feed formulation. Each variable that contributes to the optimal solution has an allowable increase and allowable decrease, as shown in Table 7. Reduced cost will indicate how much objective coefficients can be increased or decreased before the optimal solution changes. For example, the final value (in kg) of X1 feed (Corn Silage) is unaffected if the cost of corn silage increases by 2.52 or decreases by 0.92. If we increase the unit cost of X1 feed (Corn Silage) with 3 or more units, the optimal solution changes, and similarly in all other cases. The shadow prices indicate by how much the optimal solution can be enhanced or decreased by altering the values on the right-hand side (nutrient constraints) by one unit.

Table 7. Sensitivity Analysis for an Allowable Increase and Allowable Decrease of Variables that Contributes to the Optimal Solution

Feeds	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
X1	0.771	0	5	2.525	0.920
X2	0	2.991	5	1.00E+30	2.991
X3	0	2.384	5.5	1E+30	2.384
X4	1.234	0	5	1.169	3.306
X5	0	1.9	5	1E+30	1.900
X6	3.129	-1.515	16.5	1.515	1E+30
X7	1.123	0	18	4.315	1.392
X8	3.129	-2.853	17	2.853	1E+30
X9	0	4.2	18	1E+30	4.2
X10	3.129	-16.047	5	16.047	1E+30
X11	1.564	-7.208	11.5	7.208	1E+30
X12	0	11.808	30	1E+30	11.808
X13	0	27.917	45	1E+30	27.917
X14	0.983	0	18	6.883	4.545
X15	0	5.364	35	1E+30	5.364
X16	0	22.166	30	1E+30	22.166
X17	0	7.609	30	1E+30	7.609
X18	1.564	-10.818	12	10.818	1E+30
X19	1.564	-9.860	14.5	9.860	1E+30
X20	0.078	-4.084	1.7	4.084	1E+30
X21	0	25.979	37	1E+30	25.979
X22	0.2332	0	3.5	23.602	4.522
X23	0.078	-23.011	4.1	23.011	1E+30
			For Constraints		
	Final Value	**Shadow Price**	**Constraint R.H. Side**	**Allowable Increase**	**Allowable Decrease**
DM	15.64	27.11	15.64	0.6003	0.61
CP	2.091	0	2.50	1.00E+30	0.41
TDN	10.65	0	11.23	1E+30	0.57
Ca	0.21	-69.44	0.216	0.027	0.036
P	0.039	-2839.1	0.039	0.0019	0.001
ROUGH	12.51	-2.75	12.51	1.625	0.86
CONC	6.067	0	10.95	1E+30	4.88

Table 7. Continued

	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
CP	2.091	35.82	2.091	0.054	0.131
TDN	10.65	0	9.363	1.294	1E+30
ca	0.216	0	0.180	0.036	1E+30
P	0.039	0	0.0325	0.006	1E+30
ROUGH	12.517	0	4.693	7.823	1E+30
CONC	6.067	0	3.129	2.937	1E+30

The graphical representation of the two models concerning feed (in x-axis) and feed quantity (in y-axis), by all the techniques follows Figure 1 and 2. The nutrients satisfied are shown in Figure 3 and 4.

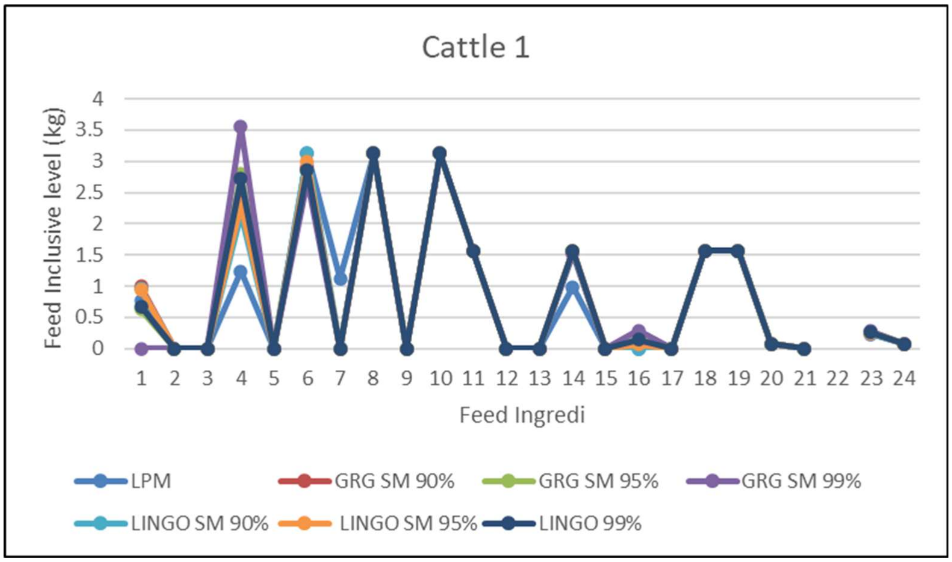


Figure 1. Graph of Feedstuff vs the feed Inclusive Level (kg) Using Three Different Techniques for Cattle 1

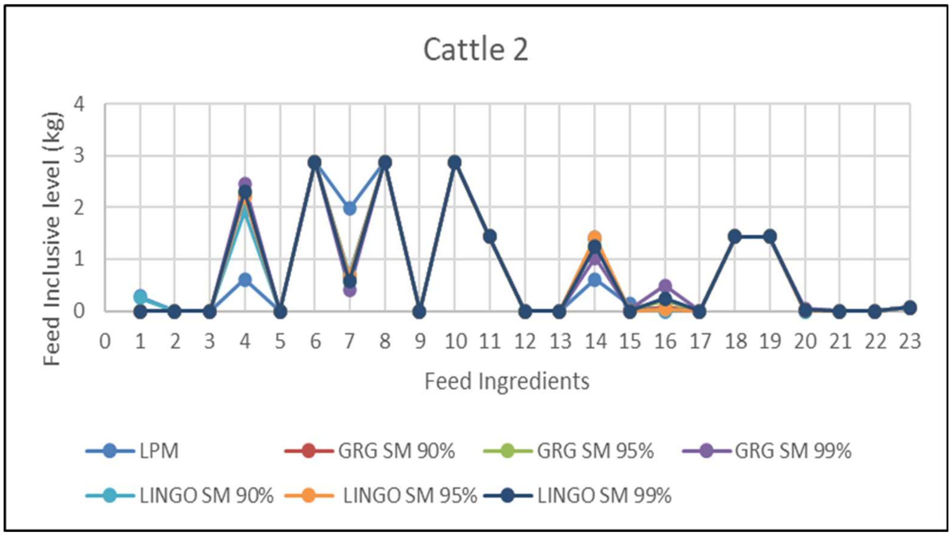
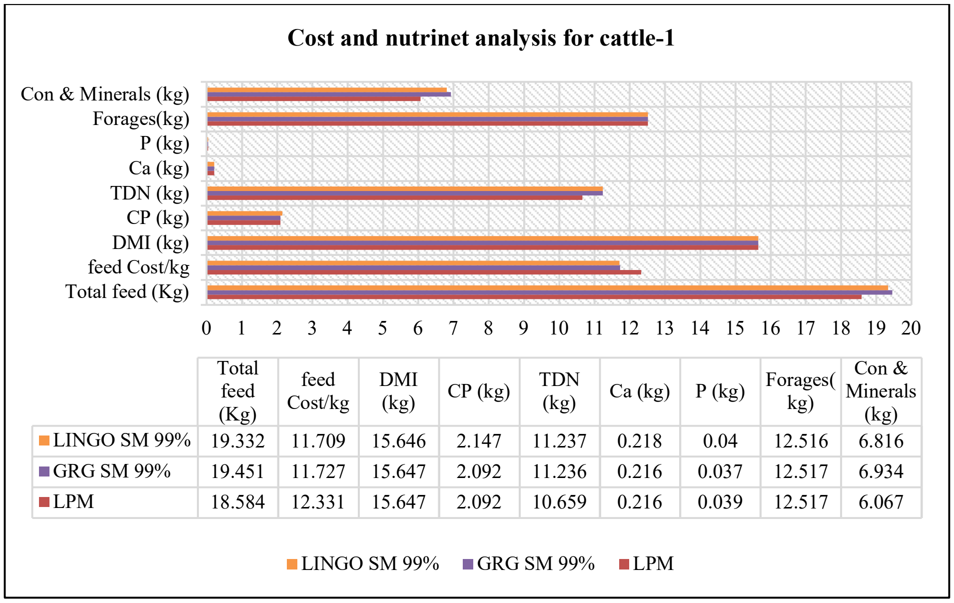
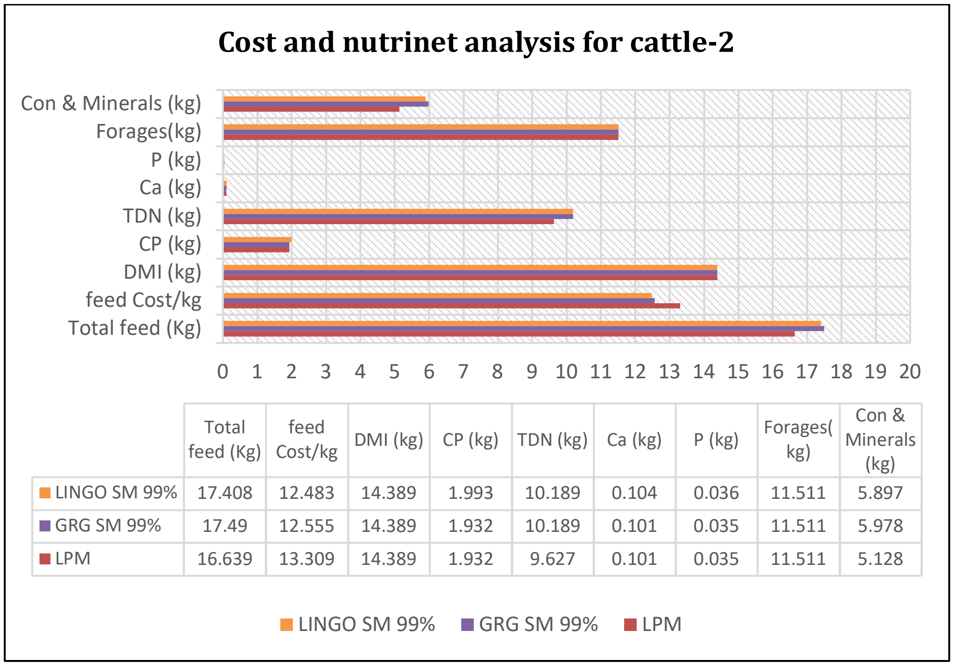


Figure 2. Graph of Feedstuff vs the Feed Inclusive Level (kg) Using Three Different Techniques for Cattle 2



	Total feed (Kg)	feed Cost/kg	DMI (kg)	CP (kg)	TDN (kg)	Ca (kg)	P (kg)	Forages(kg)	Con & Minerals (kg)
■ LINGO SM 99%	19.332	11.709	15.646	2.147	11.237	0.218	0.04	12.516	6.816
■ GRG SM 99%	19.451	11.727	15.647	2.092	11.236	0.216	0.037	12.517	6.934
■ LPM	18.584	12.331	15.647	2.092	10.659	0.216	0.039	12.517	6.067

Figure 3. Satisfied Nutrients by LP, SM for GRG and LINGO (99%) for Cattle 1



	Total feed (Kg)	feed Cost/kg	DMI (kg)	CP (kg)	TDN (kg)	Ca (kg)	P (kg)	Forages(kg)	Con & Minerals (kg)
■ LINGO SM 99%	17.408	12.483	14.389	1.993	10.189	0.104	0.036	11.511	5.897
■ GRG SM 99%	17.49	12.555	14.389	1.932	10.189	0.101	0.035	11.511	5.978
■ LPM	16.639	13.309	14.389	1.932	9.627	0.101	0.035	11.511	5.128

Figure 4. Satisfied Nutrients by LP, SM for GRG and LINGO (99%)

We can consider that Stochastic model with nonlinear constraints can be effectively used for ration formulation to find least cost feed stuffs for lactating cattle.

4. Conclusions

The present study focuses on the use of the stochastic model for the least cost- formulation of feed for lactating cattle for improved health and milk production. It has been shown that for varying nutrients such as CP, Ca & P constraints, which convert linear constraints to nonlinear, the stochastic model can be set for different confidence levels. With the assurance that 90-99 % of nutrient requirements are met in the ration, the stochastic model provides the least expense.

With rigid constraints, the stochastic model runs, while the linear model struggles with rigid constraints. Even though we have highlighted the limitations of LP model in present work, but as a deterministic approach, LP has been proved to be most effective method in least cost ration formulation in animals if all the prices and values of nutrients in feed are known as LP model guarantees optimal solution. This study has demonstrated that Stochastic programming model is better approach as compared to LP in addressing nutrient variability along with minimization of cost of feed mix in particularly lactating cattle.

Conflict of Interest

We certify that there is no conflict of interest with any financial, personal, or other relationships with other people or organization related to the material discussed in the manuscript.

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