

Randomness of Poisson Distributed Random Number in the Queue System

Ernestasia¹, Esther Nababan², Asima Manurung³

¹Technical University Delft, Delft, The Netherland

^{2,3}Faculty of Mathematic and Natural Science, Universitas Sumatera Utara, Indonesia

Abstract. In the queuing system, inter-arrival variable and service time variable are probabilistic and its pattern follow a Poisson distribution. Simulations experiment for performance measurement of a queuing system required random data. In practice, random data is built using an application program. Pseudorandom data generated from application programs often have different patterns of randomness, although in each experiment simulated the same data distribution. Level of randomness may cause the results of simulation experiments experienced statistically significant deviations, especially on problems with stochastic variables. Statistical deviation can cause errors in interpreting the results of simulation experiments, especially in the assessment of the performance of the queuing system. It is required to evaluate whether the level of randomness of pseudorandom data effect on simulation results of performance measurement of a system. Simulation experiments on a simple queuing system (M / M / 1) were carried out by using a pseudorandom number generator. Application program used to generate pseudorandom numbers is Fortran90. The experimental results show that the greater the amount of pseudorandom data, the greater the statistical deviations occur, and the smaller the degree of randomness of data. This behaviour affects the results of the simulation system in which there is a probabilistic variable that require random data to conduct simulation

Keyword: Pseudorandom, randomness, statistical error, queuing system, performance

*Corresponding author at: Faculty of Mathematic and Natural Science, Universitas Sumatera Utara Jl. Bioteknologi No.1, Padang Bulan, Kec. Medan Baru, Kota Medan, Sumatera Utara 20155

Abstrak. Dalam sistem antrian, antara variabel kedatangan dan variabel waktu pelayanan adalah probabilistik dan polanya mengikuti distribusi Poisson. Eksperimen simulasi untuk pengukuran kinerja sistem antrian diperlukan data acak. Dalam praktiknya, data acak dibangun menggunakan program aplikasi. Data pseudorandom yang dihasilkan dari program aplikasi tersebut sering memiliki pola acak yang berbeda, meskipun dalam setiap percobaan disimulasikan menggunakan distribusi data yang sama. Tingkat keacakan dapat menyebabkan hasil percobaan simulasi mengalami penyimpangan yang signifikan secara statistik, terutama pada masalah dengan variabel stokastik. Penyimpangan statistik dapat menyebabkan kesalahan dalam menafsirkan hasil eksperimen simulasi, terutama dalam penilaian kinerja sistem antrian. Hal ini diperlukan untuk mengevaluasi apakah tingkat keacakan data pseudorandom berpengaruh pada hasil simulasi pengukuran kinerja suatu sistem. Eksperimen simulasi pada sistem antrian sederhana ($M / M / 1$) dilakukan dengan menggunakan generator nomor pseudorandom. Program aplikasi yang digunakan untuk menghasilkan nomor pseudorandom adalah Fortran90. Hasil eksperimen menunjukkan bahwa semakin besar jumlah data pseudorandom, semakin besar penyimpangan statistik terjadi, dan semakin kecil tingkat keacakan data. Ini mempengaruhi hasil dari sistem simulasi di mana ada variabel probabilistik yang memerlukan data acak untuk melakukan simulasi

Kata Kunci: pseudorandom, keserampangan, kesalahan statistik, sistem antrian, performa

Received 14 July 2019 | Revised 25 July 2019 | Accepted 25 July 2019

1 Introduction

In connection with the increasingly complex real-world problems, a problem will be more easily managed and controlled if the problem is seen as a system where each component in it has a causal relationship both directly and indirectly. System performance can be evaluated by conducting simulations. In the future, simulations will be increasingly used to assess the performance of a system. This is because systems with a large number of components will be more easily evaluated by simulations.

However, the existence of uncertain variables requires the use of random numbers with appropriate data distribution [1][2]. Uniformly distributed random numbers can be generated through application programs such as Fortran, Basic, PL / 1, MS Excel. In simulating a discrete event using random data it is necessary to do a randomization test first to ensure that data can be accepted randomly following the desired data distribution, at a level or interval of confidence. Several studies regarding randomness test have shown that the data needed for the simulation is expected to be completely random [1]-[5]. The question is how much influence the truth of the " data " has on the simulation results.

The randomness of a data sequence can be seen from various sides, including from the frequency of occurrence of data at each interval class; variations in the distance between one data to the next; and patterns of back and forth or fluctuation of data. The most common methods used to test the randomness of a data are frequency test, line test (serial test), poker test (poker test), distance test (gap test), and test of increasing and decreasing run [1][4]. The selection of one method for randomness testing depends very much on problems that require random data. For example, if random data is needed which is uniformly distributed, it is enough

to do a frequency test to test randomness. But generally randomness tests are only done using one of the test kits, without doing another randomness test. The problem is, in simulating a discrete event, the resulting random variables have different patterns, even though the data distribution in each simulation is the same [3] [6] [7]. The question is whether the difference in the randomness pattern of the data in each simulation has a significant effect on the statistics χ^2 ? To answer this question, it is necessary to examine the level of randomness that was generated from each randomness test above, whether the level of randomness generated from the three randomization tests was the same. If not the same, it is necessary to examine how significant the level of difference in randomness and its effect on the simulation results.

In this study, the randomness of a data sequence generated from the pseudorandom generator is evaluated and analyzed. Random data generated from the pseudorandom generator is uniformly distributed data, which is then converted into Poisson distributed data [1][8]. The randomness of the data was tested in two stages. The first stage is to test the randomness of the data with uniform distribution, which is tested from three sides, namely frequency test, gap test and back and forth test to get an overview of the level of randomness generated by each randomness test tool. Furthermore, the generated random data is converted into Poisson-distributed random data. We use this data to simulate discrete events on simple queuing problems to produce an overview of system performance expectations. The simulation results are then analyzed to see the performance expectations of the queue system and its relationship with each level of randomness. Analysis of simulation results is expected to provide an overview of the effect of the randomization pattern of data on statistics χ^2 . Thus a conclusion and suggestion can be made regarding what steps should be done or added to each discrete event simulation process with random data to obtain more accurate results. The result of this study is can be useful other researches related to the study of randomness and other studies that require random data.

2 .Literature Review

Statistical errors in the simulation results are generally measured by the interval of expectation that contain unexpected values. Of about 50% of all publications regarding simulation studies, only 23% of the simulation results are credible information that includes statistical analysis of simulation results [9].

In its implementation in stochastic simulations, the width of the interval or interval of confidence will be smaller along with the amount of data collected. To overcome this there are two scenarios. The first scenario is to add the length of the simulation experiment as an input parameter to the model. This method is based on the argument that the more the number of simulations is carried out, the better the results, and statistical errors that occur are accidental factors [10].

The concept of randomization is considered as a special case of the epistemological concept of an unpredictable process. Eagle gave an explanation of the concept of intuitive randomization, suggesting that the understanding of randomness so far is no longer true completely. Hence, a more understanding and philosophical study of randomness is required. Throughout the history of producing random numbers, there are rows of random numbers obtained from several sources of random numbers "pseudorandom generator", after being tested their randomness shows that the sequence of random numbers produced is apparently not very random, depending on the type of randomness test [11].

2.1. Pseudorandom Randomization Problems

Random numbers generated through the current application program are not really random, but rather are pseudorandom random numbers whose level of randomness in an area of trust is sufficient for the user. The pseudorandom process is a process that appears to be random but actually it is not random. The pseudorandom sequence specifically shows statistical randomness when it is produced by a causal process which is a deterministic process as a whole. Such process is an easier method to produce pseudorandom numbers than other methods [5][7][8]. The advantage is that random numbers generated can be repeated again with the exact same random number results. This behaviour is considered useful to be used as part of software test cases. To produce actual random numbers, it is necessary to measure systems that are accurate and repeatable from a process that is truly non-deterministic.

In practice, the algorithmic generators of pseudorandom uniformly distributed numbers (PRNG) are generally used to describe randomness in stochastic simulations. The basis of the PRNG theory has long been found for example by Knuth 1998, and for the last 50 years, various algorithms capable of producing pseudorandom random numbers have been. Technically, all random numbers generators are equivalent to PRNG (LC-PRNGs) in producing a periodic sequence of numbers. One of them is a recursive algorithm in integer modulo M [9]. Whereas for 32-bit computers, the multiplication of LC-PRNG with modulus $2^{31}-1$ is recommended as an acceptable source for randomization modeling [3][8]. There are several generators of random numbers that are widely used in modern computer such as GPSS (version H and PC), SIMSCRIPT II.5, SIMAN and SLAM II [12]. However, generating pseudorandom numbers in real case scenario may raise multiple problems. Therefore the quality of producing random numbers and random numbers generated for discrete event simulations needs to be evaluated. Evaluation can be done by conducting a study of the results of numerical experiments on discrete event simulations using pseudorandom random numbers.

2. 2 Effect of Data Randomization on Simulation Results

Stochastic simulation or discrete event simulation should be seen as a statistical simulation experiment so that analysis of output data is a necessary condition for credible final results. There are several tools for conducting simulations, including random number generator. After the design of the simulation model is valid, and the model has been implemented and verified, researchers continue to face problems regarding output analysis [4][11]. As another scientific paradigm, the output of simulation experiments must be accepted on a fairly small error. If not, statistical errors can produce non-credible conclusions.

3 Research Methodology

In this study, we conduct a simulation was conducted to estimate the performance of the queue system, where customers arrival data on the arrival of customers is built using with pseudorandom numbers with Poisson distribution was constructed from pseudorandom numbers. The tool used for simulation is the FORTRAN application program for simulation programs and generating pseudorandom numbers; MS Excel to do some calculations on randomness tests; and table χ^2 for randomness test data and statistical theory to assess statistical deviations and so on, and QM (Quantitative Management) application programs to calculate the performance of the queue system.

As our test case scenario, we develop a system that simulate a mini market with 1 cash register. Sung a simple queue system model (M / M / 1) where M represents the average arrival of customers, M represents the service level, and 1 in the service facilities in the system or one channel.

Distribution of potential customer arrivals time follows the Poisson distribution. Service is set to follow the First Come First Serve rule. We use a single channel service in our scenario. Service distribution follows a Poisson distribution ($\lambda < \mu$). System capacity is assumed to be unlimited, and there is no rejection. Simulations are carried out in a simple (M / M / 1) queuing system, such as a cash register at a supermarket. At the supermarket the number of lanes is single, the level of customer arrivals is Poisson distribution, the service time is exponentially distributed, and the queue size is unlimited. The queue system follows the First In First Out rule.

Notation :

- N = number of customers in the system
- Pn = the certainty probability of the customer in the system
- λ = the average number of customers come per unit time
- μ = the average number of customers served per unit time

- Po = probability of no customer in system
 P = level of intensity of service facilities
 L = average number of customers expected in the system
 Lq = expected number of customers waiting in the system
 W = the time expected by customer while in the system
 Wq = the time expected by customer while waiting inside queue
 $1 / \mu$ = average service time
 $1 / \lambda$ = average time between arrivals
 S = number of service facilities

Intensity or Performance = $\rho = \lambda / \mu < 1.0$

Poisson distribution for simple queue problems :

$$F_i = \sum_{i=b}^{\infty} \frac{\alpha^{x_i} e^{-\alpha}}{x_i!} \quad (1)$$

where F_i = Poisson probability on the i -th category ; $e = 2.7182$

Simulation is done by building random numbers for customer arrival rates. The number of random numbers used varies, namely 100, 500, 1000, 5000, 7500, and 10000. The random numbers generated are then tested for randomness by carrying out statistical tests, namely frequency test, gap test, and forward test (Run test). Analysis of number randomness based on randomness test results.

For Poisson distribution data in this study, the testing phase is:

a. Randomness test

Hypothesis:

H0: Pseudorandom data is random

H1: Pseudorandom data is not random

Statistical tests:

$$z = \frac{r - \left\{ \frac{2(n_1 n_2)}{n_1 + n_2} \right\} + 1}{\sqrt{\frac{2n_1 n_2 (2n_1 n_2 - n_1 n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}}} \quad (2)$$

Rejection area : Reject Ho, if $Z_{hit} > Z_{\alpha/2}$

b. Test on Poisson Distribution

H0: Pseudorandom data is Poisson distribution

H1: Pseudorandom data is not Poisson distribution

Statistical tests : $D = \left[\text{Sup} \right] \int_x |S(x) - F_0(x)|$

Rejected area : Reject H_0 , if $D_{hit} > D(1-\alpha, n)$

α = the average of the number of arrivals

X_i = the number of the i -th arrival per unit time

X_a = upper limit ; X_b = lower limit

The next stage is calculating expectations of system performance, and calculating standard deviations or statistical deviations to obtain conclusions regarding estimates of performance.

4. Result and Discussion

The queuing system model used in this study is $M/M/1$. The server intensity level is obtained from the average number of customers expected in the system and the number of customers expected to wait in the queue, the time expected by each customer waiting for service, and the time expected by each customer to wait in the queue. The probability that the server is busy, namely the probability of a customer having to wait, also called the utilization factor or performance measure is $\frac{\mu}{\lambda}$, which is the ratio between the level of arrival and the level of service.

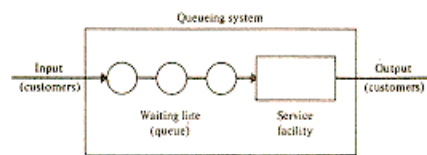


Figure 1. Queuing System (M/M/1)

Analysis of simulation results by comparing the results of the randomness and Poisson distribution test:

- Transient and steady-state characteristics of the stochastic process
- Statistical analysis for steady-state parameters.
- Measurement of system performance.

The chi-square statistic is used to determine how well the set observation can be represented by a given distribution, where each observation is located in one of the k different categories. If the number of events is preserved is O_i , and the expected number of events / events is E_i known for each category, then $\alpha / 2$ statistics can be determined. The calculation results (table 1) shows that χ^2 observations do not exceed χ^2_{table} , then the hypothesis that the random numbers generated are truly random, can be accepted at a rejection rate of 5%.

Table 1. The value of χ^2 , $\chi^2_{\leq 5\%}$, $\chi^2_{\leq 95\%}$

N	χ^2	Freq test	Gap test	Run test
100	χ^2_{obs}	6.3	12.46	0.89
	$\chi^2_{\leq 5\%}$	17.33	21.13	9.448
	$\chi^2_{\leq 95\%}$	4.575	5.224	0.697
500	χ^2_{obs}	24.7	38.9	1.797
	$\chi^2_{\leq 5\%}$	18.3	43.77	11.07
	$\chi^2_{\leq 95\%}$	5.575	18.47	1.114
1000	χ^2_{obs}	42.1	14.33	2.78
	$\chi^2_{\leq 5\%}$	18.21	21.04	9.55
	$\chi^2_{\leq 95\%}$	4.565	5.33	0.72
5000	χ^2_{obs}	238.22	433.2	17.88
	$\chi^2_{\leq 5\%}$	18.33	67.5	11.87
	$\chi^2_{\leq 95\%}$	4.57	33.97	1.155
7500	χ^2_{obs}	342.4	778.8	20.21
	$\chi^2_{\leq 5\%}$	12.59	67.8	9.447
	$\chi^2_{\leq 95\%}$	4.525	36.8	2.944
10000	χ^2_{obs}	4554	958.223	39.12
	$\chi^2_{\leq 5\%}$	18.31	67.8	16.51
	$\chi^2_{\leq 95\%}$	4.565	34.77	2.744

Likewise, chi-square observation (χ^2_{obs}) in the three statistical tests for rejection rates is 95% greater than χ^2_{table} , particularly randomly spaced, because χ^2 of gap test is much greater than χ^2_{table} . Then for $N = 100$, the hypothesis is that random data is significantly acceptable, even though for frequency distribution and back and forth patterns, χ^2 is not significantly greater than χ^2_{table} .

This can be caused by the spread of data that is not equally distributed at each interval class in the frequency test. Data distribution in each interval class is not evenly distributed, with a standard deviation that is quite large, 8.59. It can be seen that the pattern of ups and downs of data is not really randomly uniform. The number of ups and downs of the data is as follows:

Variations between $n = 1$, with $n = 2, 3$ and 4 are quite spaced, and not uniform. The value of randomness will be more significant if the amount of On is distributed evenly in each category. Significant random data hypotheses were accepted if $\chi^2_{\text{table}} (P = 95\%) < \chi^2_{\text{obs}} < \chi^2_{\text{table}} (P = 5\%)$. The results of the frequency test, at the chance of rejection of 5%, the value of χ^2 of the observation results is smaller than χ^2_{table} , so the hypothesis that random data is rejected. Whereas in the probability of rejection χ^2_{table} of hypotheses are accepted because χ^2_{obs} are greater than χ^2_{table} .

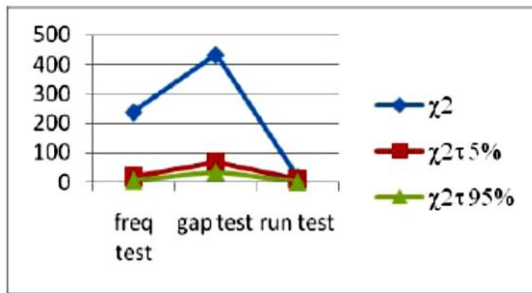


Figure 2. Plot χ^2 and χ^2_{table} for $n = 500$

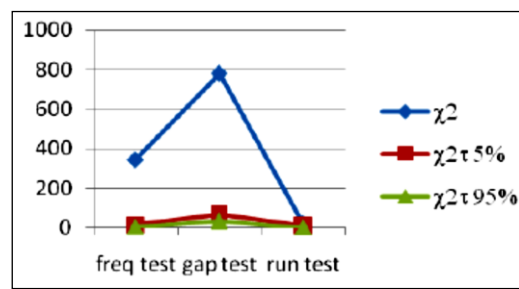


Figure 3. Plot χ^2 and χ^2_{table} for $n = 1000$

The results of the back and forth test, on the chance of rejection of 5%, the value of χ^2 of the observation is smaller than χ^2_{table} , then the hypothesis that random data at the chance of rejection is 5% is accepted. Whereas at the opportunity of rejecting 95% the hypothesis that random data is accepted is not significant.

For the number of random numbers $N = 1000$, the results of observations in the three statistical tests are as follows:

Frequency test shows that at the 5% probability of rejection, the hypothesis is rejected; while the odds of rejection are 95% accepted. Then it can be concluded that the hypothesis that the data distribution is random is rejected. From the table it can be seen that in each interval class, the data is not evenly distributed, with a standard deviation of 7.87. The gap test shows that χ^2_{obs} are between χ^2_{table} ($P = 95\%$) and χ^2_{table} ($P = 5\%$). Similarly, the back and forth test shows that χ^2_{obs} are between χ^2_{table} ($P = 95\%$) and χ^2_{table} ($P = 5\%$).

The hypothesis estimates for a 5% probability, the hypothesis is rejected, while at 95% probability, the hypothesis that the pseudorandom data is randomly distributed, can be accepted significantly. Results that can accept hypotheses at intervals, probabilities, and retention between 5% and 95% and pattern gaps even though for frequency distribution, with standard deviation 8, the hypothesis cannot be accepted significantly at 95% or a 5% chance.

Statistical tests for the number of random numbers $N = 7500$, the statistical test hypothesis for the estimate of 5%, the hypothesis that random data is rejected, while at the 95% chance of debate, the hypothesis that random data can be accepted significantly. As for the larger amount of data, namely $N = 10,000$, the statistical test hypothesis for debate is 5%, the hypothesis about random data is rejected, while at the 95% chance of debate, the hypothesis that random data can be accepted significantly. For large numbers 5000, 7500 and 10000, it turns out that the frequency test indicates a profit of 5%, the hypothesis of random data is rejected, while at a profit of 95%, the hypothesis that random data can be accepted significantly. Data distribution frequency of occurrence of data is not a random distribution uniform distribution. This uniform distribution of pseudorandom data is then used to create Poisson distribution pseudorandom numbers

The next statistical test is carried out to test whether the sequence of numbers is Poisson distribution. The probability distribution of inter-arrival times and the probability distribution of service time in the queuing system (M / M / 1) used are:

Table 2. Inter-Arrival Time Probability Distribution

Probability	Upper limit	Lower Limit	Inter arrival time (minutes)
0.4	0	0.4	1
0.3	0.4	0.7	3
0.1	0.7	0.8	5
0.2	0.8	1	10

Table 3. Probability Distribution of the Service Time

Probability	Lower Limit	Upper Limit	Service Time (min)
0.3	0	0.3	3
0.35	0.3	0.65	6
0.35	0.65	1	9

The inter-arrival time and service time is Poisson distribution pseudorandom data. The number of customers that come is $n = 15$. The results of the 1 simulation are as follows:

Table 4. Simulation of queuing system (M/M/1) with $n = 15$

Cust	Inter arrival time (minutes)	Service Time (minutes)	Waiting Time (minutes)	Total Time
1	1	3	-	3
2	1	6	2	8
3	3	6	5	11
4	5	6	6	12
5	5	6	7	13
6	1	6	12	18
7	10	9	8	17
8	3	9	14	23
9	1	3	22	25
10	5	3	20	23
11	3	3	20	23
12	10	3	13	16
13	1	6	15	21
14	1	9	20	29
15	3	6	26	32

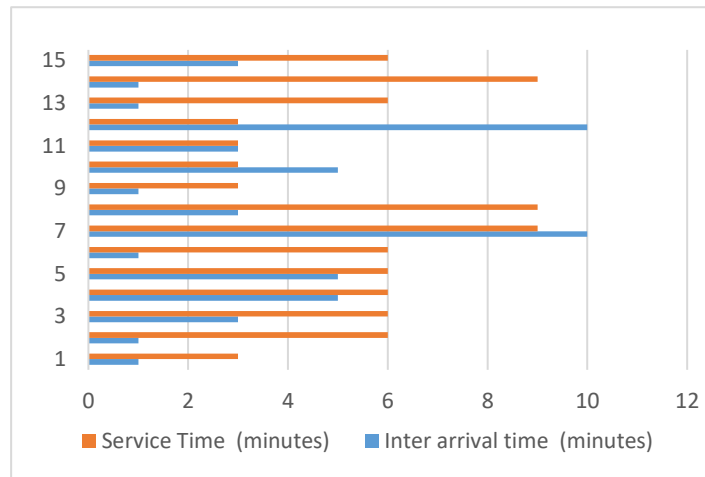


Figure. 3. The graph between inter-arrival time and service time

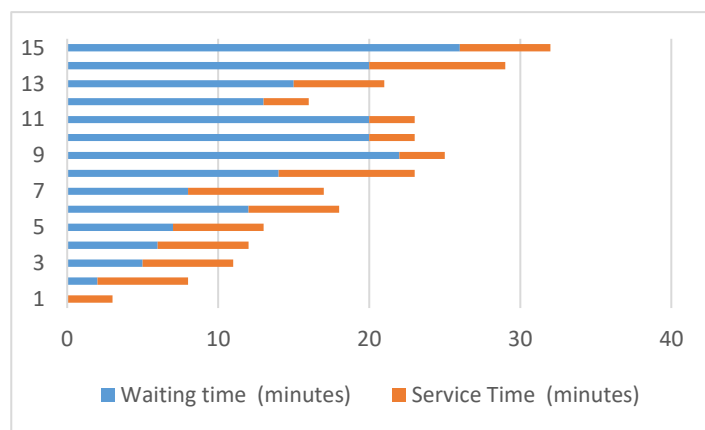


Figure. 4. The graph between waiting time and service time

The performance expectation of the queue system above is 0.76. We also perform the simulation with a larger amount of data: 100, 500, 1000, 7500, and 10.000 customers. The statistical test of pseudorandom data with Poisson distribution for the amount of data = 100 indicates that D_{hit} (D calculated) is greater than $D_{(1-\alpha, n)}$ or $D_{hit} > D_{(1-\alpha, n)}$. Thus the hypothesis that the pseudorandom data with Poisson distribution is rejected. On the contrary, the difference in distance between $D_{hit} > D_{(1-\alpha, n)}$ is not significant.

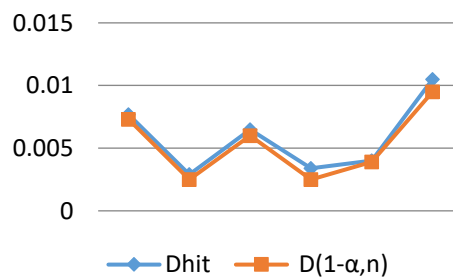


Figure. 5. The plot of Poisson distribution tests (M/M/1) n=100

The same thing happened in the simulation with the number of pseudorandom data $n = 500$. However in the simulation with the number of pseudorandom data $n = 1000$, the hypothesis of Poisson distribution data was received, and the distance between D_{hit} and $D(1-\alpha, n)$ was quite significant.

$N = 500$, the same thing happened in the simulation of pseudorandom data. $N = 1000$, the hypothesis of data is Poisson distribution was not rejected, and the distance between D_{hit} and $D(1-\alpha, n)$ was quite significant.

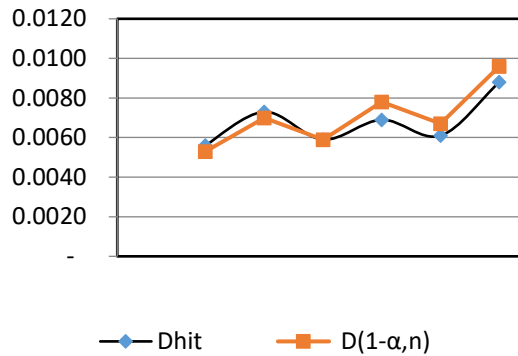


Figure. 6. Plot of Poisson distribution tests (M/M/1) $n=1000$

Next, the simulation with a number of 'run' varies, namely $s = 100, 500, 1000,$ and 5000 . The number of pseudorandom data is $n = 100$. Performance expectations are obtained from the transient phase or steady state conditions from the performance graph as shown below.

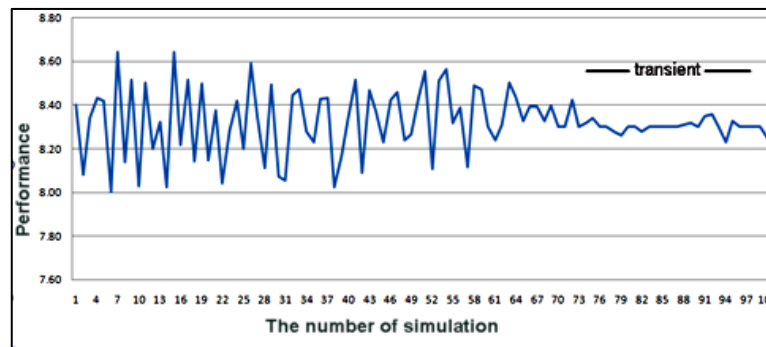


Figure. 7. The graph of system performance ($s = 100$)

For 100 run of simulation ($s=100$), transient phase started at the 70th to the 93rd run, with the performance expectation is around 0.83. The result of the performance expectation and Poisson distribution tests on each number of run can be seen at table 5, 6, and 7.

Table 5. Performance Expectation (n=100)

The number of runs s	Performance Expectation	Tests of Poisson distribution	
		D_{cal}	$D_{(1-\alpha,n)}$
100	0.83	0.0078	0.0084
500	0.66	0.0029	0.0035
1000	0.74	0.0040	0.0049
5000	0.82	0.0087	0.0096
7500	0.88	0.0040	0.0044
10000	0.84	0.0082	0.0088

Table 6. Performance expectation (n=500)

The number of runs s	Performance Expectation	Tests of Poisson distribution	
		D_{cal}	$D_{(1-\alpha,n)}$
100	0.95	0.0077	0.0079
500	0.76	0.0029	0.0031
1000	0.99	0.0065	0.0064
5000	0.63	0.0034	0.0037
7500	0.88	0.0040	0.0041
10000	0.78	0.0105	0.0100

Table 7. Performance expectation (n=1000)

The number of runs s	Performance Expectation	Tests of Poisson distribution	
		D_{cal}	$D_{(1-\alpha,n)}$
100	0.97	0.0081	0.0083
500	0.81	0.0024	0.0025
1000	0.99	0.0059	0.0060
5000	0.87	0.0033	0.0035
7500	0.98	0.0049	0.0049
10000	0.67	0.0121	0.0106

Performance comparisons based on the number of pseudorandom data and the number of pseudorandom data :

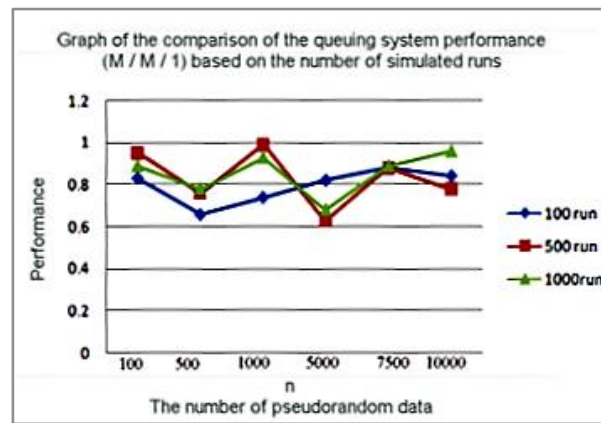


Figure 8. The performance comparison based on the number of run.

In our program, we construct random numbers using a pseudorandom generators with Poisson distribution. Subprogram to build random number variations with uniform distribution of $0 < u_i < 1$ based on the power residue method. With the power residue method, a repetition program occurs in each period of rows of random numbers generated. This is one of the causes of the randomness of the data hypothesis in the chance of rejection which is quite small, ie 5% is rejected for the amount of data that is getting bigger.

Likewise, the opportunity for data rejection is quite large, ie 95% of the randomness hypothesis of the data is received. On the number of data $n = 1000$, the value of system performance for the number of run $s = 500$ and the number of run $s = 1000$ is close to 1. But in the larger amount of data, namely 5000, 7500, and 10000, the performance value is below 0.9. Even in the amount of data = 5000, the average value of performance is below 0.8. This shows that the greater the random data generated, the greater the chance that the chance of randomization of the data produced will be smaller and more insignificant. The level of randomness of this data proved to have an effect on the simulation results on the M / M / 1 queue problem.

The simulation results show that the randomness level of the data hypothesis can be accepted as a random number at the rejection level of 5% or at the rejection level of 10%. While the Poisson distribution test shows that the more data, the greater the possibility of data not being Poisson distribution.

Based on our observation, we conclude that there are two possibilities that cause this. The first one is that in a period of random number sequences, the data experience repetition, and the repetition that occurs does not fully repeat the total of each random sequence.. The pattern of data distribution generated by random number generator application programs depends on the method used to generate random numbers. Repetition of rows of data for a certain period is a result of the method of generating random numbers used in the application program. In the Fortran application program, subprograms to produce random number variations have a uniform distribution of $0 < u_i < 1$ based on the deterministic power residue method so that a repetition

period will occur in a data line. The second possibility is that the simple queuing system ($M / M / 1$) does not require large pseudorandom data because one server in a period of time is only capable of serving a limited number of customers. A Large pseudorandom data does not comply with the test case that uses a single channel service.

5. Conclusions

In using pseudorandom random numbers, it is necessary to consider software applications and methods used as generators of pseudorandom numbers. This is needed to determine the length of the repetition period of the data. The application program used to generate random numbers affects the randomness of the sequence of pseudorandom random numbers generated, which will also have an impact on the simulation results to measure the performance of a system.

In conducting a simulation experiment with pseudorandom numbers, it is necessary to consider the number of pseudorandom numbers needed, and the application software used to generate pseudorandom numbers. In the queue system ($M / M / 1$), for the small number of customer arrivals, the level of randomness does not have a significant effect on system performance, as well as a large amount of pseudorandom data. This is likely related to the type of queue system being reviewed. To measure the performance of a simple queue system ($M / M / 1$) the amount of pseudorandom data used does not need to be very large. On the other hand for multi-server queuing systems, large numbers of pseudorandom numbers are needed. Further research can be done to test randomness and its effect on other queuing systems

REFERENCES

- [1] B.S. Gottfrieds. *Elements of Stochastic Process Simulation*, Prentice Hall Inc. NJ., 1992
- [2] S.J. Kim, S., K. Umeno, and A. Hasegawa, "On the NIST Statistical Test Suite for Randomness" in *IEICE Technical Report*, vol.103, no. 449, pp. 21-27. 2003
- [3] P. L'Ecuyer, "Uniform Random Number Generators" in *Proceedings of the. 1998 Winter Simulation Conference. WSC'98*. Washington: pp 97-104. 1998
- [4] H. Niederreiter and L. Kuipers, *Uniform distribution of sequences*, Dover Pub., 2005
- [5] W. Dembsky. 2008, "Randomness by Design", <http://www.leaderu.com/offices/dembski/docs/bd-random.html> (Accessed: July. 8, 2014).
- [6] K. Park and K. W. Miller, "Random Number Generators: Good Ones are Hard to Find". *Communication of the ACM*, vol.31, no.10, pp.1192-1201. 1988.
- [7] O'Donnel, C., G. Edward Suh, dan Srinivas Devadas, "PUF Based Random Number Generation" MIT CSAIL CSG Technical Memo, pp. 481, 2005.
- [8] U.K.M Maheswari, R. Kundu, and H. Saxena, Recent Trends in Engineering and Applied Mathematics, "*International Journal of Pure and Applied Mathematics*" vol.118, no. 22 pp 331-336. 2018.
- [9] K. Pawlikowski, "Steady-State Simulation of Queueing Processes: a Survey of Problems and Solutions". *ACM Computing Surveys*: pp 123-170. 1990.
- [10] Eagle, A, "Randomness is unpredictable," *British Journal of Philosophical Science* 56 pp 749-790. 2005.
- [11] O. Goldreich, *Computational Complexity: A Conceptual Perspective*. Cambridge Univ. Press ,2008.
- [12] A. M Law and W. D. Kelton. *Simulation Modelling and Analysis*. McGraw-Hill, 1991