SOLVING DUAL FULLY FUZZY LINEAR SYSTEM BY USE FACTORIZATIONS OF THE COEFFICIENT MATRIX FOR TRAPEZOIDAL FUZZY NUMBER

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Abstract. This article discusses the solution for a dual fully fuzzy linear system in the form of 
\[ \tilde{A} \tilde{x} \oplus \tilde{b} = \tilde{B} \otimes \hat{x} \oplus \tilde{d} \]

is then transformed into the form 
\[ \tilde{A} \otimes \hat{x} = \tilde{B} \otimes \hat{x} \oplus \tilde{f} \]

Then the coefficients of the matrix \( \tilde{A} \) and \( \tilde{C} \) each of us form into a factorization of \( LU \) and then we will construct a simple algorithm to solve the system.

1. INTRODUCTION

the development of the fuzzy concept analysis has been written by several authors, among them [16], in this paper the author will focus on the fully fuzzy linear equation system. Many methods have been found to determine the solution of a system of fuzzy linear equations \( \tilde{A} \otimes \hat{x} = \tilde{b} \), fully fuzzy linear equation system \( \tilde{A} \otimes \tilde{x} = \tilde{b} \) and dual fully fuzzy linear system \( \tilde{A} \otimes \tilde{x} \oplus \tilde{b} = \tilde{B} \otimes \hat{x} \oplus \tilde{d} \), some of which are at the solving of the system of fuzzy linear equations by [6] and [7]. And also to solving of the fully fuzzy linear equation system by [15] and [11]. Next for the completion of the dual fully fuzzy linear equation system discussed by [1] in 2010 with triangular
fuzzy number. In this article we will discuss the development of the methods discussed [1] that is for trapezoidal fuzzy number. The final part will be given a solution example as an illustration of the method given.

2. TRAPEZOIDAL FUZZY NUMBERS

Some basic definitions have been explained by [1] in his research. Here are some definitions of these fuzzy numbers:

**Definition 1.1** A fuzzy number is a fuzzy set \( \tilde{U} : R \rightarrow [0,1] \) which satisfies the following:

1. \( u \) is upper semi-continuous.
2. \( u(x) = 0 \) outside the interval \([c,d]\).
3. there exist real numbers \( a, b \) in \([c,d]\) such that,
   
   (i) \( \tilde{u}(x) \) monotonic increasing in \([c,a]\).
   
   (ii) \( \tilde{u}(x) \) monotonic the decreasing in \([b,d]\).
   
   (iii) \( \tilde{u}(x) = 1 \) for \( a \leq x \leq b \).

**Definition 1.2** A fuzzy number \( \tilde{U} \) dalam R is a pair \((\underline{U}, \overline{U})\) which satisfies the following:

1. \( \underline{U} \) is a bounded left continuous non decreasing function over \([0,1]\).
2. \( \overline{U} \) is a bounded left continuous non increasing function over \([0,1]\).
3. \( \underline{U}(r) \leq \overline{U}(r), 0 \leq r \leq 1 \).

The form of the function of the fuzzy trapezoidal number is:

\[
\mu_{\tilde{U}}(X) = \begin{cases} 
\frac{(x-a)}{(m-a)} & \alpha \leq x \leq m, \\
1 & m \leq x \leq n, \\
\frac{(\beta-x)}{n-m} & n \leq x \leq \beta, \\
0 & \text{lainnya.}
\end{cases}
\]  

(1)

In this study, fuzzy numbers are written in the form \( \tilde{u} = (a, b, \alpha, \beta) \), with \( a, b \) is the center, \( \alpha \) is the left width and \( \beta \) is the right width, for
example \( \tilde{u} = (a, b, \alpha, \beta) \) as \( \tilde{u} = (6, 4, 3, 2) \). For arbitrary fuzzy number \( \tilde{u} = (a, b, \alpha, \beta) \), the form of the membership function is

\[
\mu_{\tilde{u}}(X) = \mu_{\tilde{u}}(m, n, \alpha, \beta) = \begin{cases} 
1 - \frac{(m - x)}{\alpha} & m - \alpha \leq x \leq m, \alpha > 0, \\
1 & m < x < n, \\
1 - \frac{(x - n)}{\beta} & n \leq x \leq n + \beta, \beta > 0, \\
0 & \text{otherwise.}
\end{cases}
\]

Parametric functions of fuzzy numbers \( \tilde{u} = [\underline{u}(r), \overline{u}(r)] \) can be written into form:

\[
\underline{u}(r) = a - (1 - r)\alpha \quad \text{and} \quad \overline{u}(r) = b + (1 - r)\beta
\]

**Definition 1.3**: For fuzzy number \( \tilde{u} = (\underline{u}(r), \overline{u}(r)), \tilde{v} = (\underline{v}(r), \overline{v}(r)) \) and real number \( k \) can be defined:

1. \( \tilde{u} = \tilde{v} \) if and only if \( \underline{u}(r) = \underline{v}(r) \),
2. \( \tilde{u} \oplus \tilde{v} = \tilde{u} = (\underline{u}(r) + \underline{v}(r), \overline{v}(r) + \overline{u}(r)) \),
3. \( \tilde{u} \ominus \tilde{v} = \tilde{u} = (\underline{u}(r) + \overline{v}(r), \overline{u}(r) + \underline{v}(r)) \),
4. \( \tilde{u} \otimes \tilde{v} = [\min (\underline{u}(r)\underline{v}(r), \underline{u}(r)\overline{v}(r), \overline{u}(r)\underline{v}(r), \overline{u}(r)\overline{v}(r)), \\
\max (\underline{u}(r)\underline{v}(r), \underline{u}(r)\overline{v}(r), \overline{u}(r)\underline{v}(r), \overline{u}(r)\overline{v}(r))] \),
   the product of two parametric fuzzy numbers
   (i) for \( \tilde{u} \geq 0 \) and \( \tilde{v} \geq 0 \), then \( \tilde{u} \otimes \tilde{v} = (\underline{uv}, \overline{uv}) \).
   (ii) for \( \tilde{u} \geq 0 \) and \( \tilde{v} \leq 0 \), then \( \tilde{u} \otimes \tilde{v} = (\underline{uv}, \overline{uv}) \)
5. Scalar multiplication

\[
k\tilde{u} = \begin{cases} 
(k\underline{u}(r), k\overline{u}(r)), k \geq 0, \\
(k\overline{u}(r), k\underline{u}(r)), k \leq 0.
\end{cases}
\]

**3. Algebra of Trapezoidal Fuzzy Number**

In this section, we will discuss arithmetic for fuzzy trapezoidal numbers. In this discussion, two different fuzzy trapezoid numbers will be given, \( \tilde{u} = \)}
(a, b, α, β) and \( \tilde{v} = (c, d, γ, δ) \). Then the fuzzy trapezoid chain will be converted into parametric as follows: \( \tilde{u}(r) = a - (1-r)α \) and \( \tilde{u}(r) = b + (1-r)β \), and \( \tilde{v}(r) = c - (1-r)γ \) and \( \tilde{v}(r) = d + (1-r)δ \).

The following will be given a new definition for positive trapezoid fuzzy numbers and negative trapezoid fuzzy number based on trapezoidal area on positive and negative axes.

**Definition 1.4** a fuzzy number \( \tilde{A} = (a, b, α, β) \) is said to be positive (negative) if:

1. a trapezoidal fuzzy number \( \tilde{A} = (a, b, α, β) \) is said to be positive if \( a - α \geq 0 \), and a trapezoidal fuzzy number \( \tilde{A} = (a, b, α, β) \) is said to be positive if \( b - β \leq 0 \);

2. if \( a - α \leq 0 \), then there are 3 cases:
   - if \( a, b \geq 0 \), then \( \tilde{A} \) called positive fuzzy numbers if \( a + b - \frac{α}{2} + \frac{β}{2} \geq 0 \), and \( \tilde{A} \) called negative fuzzy numbers if \( a + b - \frac{α}{2} - \frac{β}{2} < 0 \);
   - if \( a \leq 0 \) and \( b \geq 0 \), then \( \tilde{A} \) called positive fuzzy numbers if \( a + b - \frac{α}{2} + \frac{β}{2} \geq 0 \), and \( \tilde{A} \) called negative fuzzy numbers if \( a + b - \frac{α}{2} + \frac{β}{2} < 0 \);
   - if \( a, b \leq 0 \) dan \( b + β \geq 0 \), then \( \tilde{A} \) called positive fuzzy numbers if \( a + b - \frac{α}{2} + \frac{β}{2} \geq 0 \), and \( \tilde{A} \) called negative fuzzy numbers if \( a + b - \frac{α}{2} + \frac{β}{2} < 0 \).

Then for the arithmetic algebraic process the two trapezoid fuzzy numbers are as follows:

1. Penjumlahan
   \[
   \tilde{w}_1 = \tilde{u} \oplus \tilde{v} = [(a - (1-r)α) + (c - (1-r)γ), b + (1-r)β + d + (1-r)δ], \]
   \[
   b + d + (1-r)(β + δ) = a + c, b + d, α + γ, β + δ. \]

2. Reduction
   \[
   \tilde{w}_2 = \tilde{u} \ominus \tilde{v} = [(a - (1-r)α) - (d + (1-r)δ), (b + (1-r)β) - (c + (1-r)γ)], \]
   \[
   (b - c) + (1-r)(β + γ) = (a - d), (b - c), (α + δ), (β + γ). \]
On the other hand Amit Kumar et al. [13] determine inverse of Trapezoid fuzzy numbers are:

$$\tilde{u} = -(a, b, \alpha, \beta) = (-b, -a, \beta, \alpha).$$  \hspace{1cm} (5)

From (3) we have that, for every trapezoidal fuzzy number  $$\tilde{u} = (a, b, \alpha, \beta)$$ there is a trapezoidal fuzzy number  $$\tilde{v} = (-a, -b, -\alpha, -\beta)$$ such that  $$\tilde{u} \oplus \tilde{v} = \tilde{0} = (0, 0, 0).$$ however from (4) we have that for every trapezoidal fuzzy number  $$\tilde{u} = (a, b, \alpha, \beta)$$ there is a triangular fuzzy number  $$\tilde{v} = (b, a, -\beta, \alpha)$$ such that  $$\tilde{u} \ominus \tilde{v} = \tilde{0} = (0, 0, 0).$$ Now we can see that  $$\tilde{v} = (-a, -b, -\alpha, -\beta) = -(b, a, -\beta, -\alpha) = -\tilde{v}

3. Scalar product as 

$$\lambda A = \begin{cases} 
(\lambda m, \lambda n, \lambda \alpha, \lambda \beta) & \lambda \geq 0, \\
(\lambda m, \lambda n, -\lambda \beta, -\lambda \alpha) & \lambda < 0.
\end{cases}$$ \hspace{1cm} (6)

4. Multiplication for two fuzzy numbers  $$\tilde{u} = (\underline{u}(r), \overline{u}(r))$$ and  $$\tilde{v} = (\underline{v}(r), \overline{v}(r))$$ for  $$r \in [0,1]$$ can be written as follows:

- if  $$\tilde{u}$$ positive fuzzy numbers and  $$\tilde{v}$$ positive fuzzy number, than:

$$\begin{cases} 
\underline{w}(r) = \underline{u}(r)\underline{v}(1) + \underline{u}(1)\underline{v}(r) - \underline{u}(1)\underline{v}(1), \\
\overline{w}(r) = \overline{u}(r)\overline{v}(1) + \overline{u}(1)\overline{v}(r) - \overline{u}(1)\overline{v}(1). 
\end{cases}$$ \hspace{1cm} (7)

let

$$\begin{align*}
\tilde{u} &= [\underline{u}(r), \overline{u}(r)] = [a - (1 - r)\alpha, b + (1 - r)\beta], \\
\tilde{v} &= [\underline{v}(r), \overline{v}(r)] = [c - (1 - r)\gamma, d + (1 - r)\delta].
\end{align*}$$

then the center  $$\tilde{w}$$ is  $$w_0 = abcd.$$ from formula (7) we have  $$\tilde{w} = \tilde{u} \otimes \tilde{v} = (\underline{w}(r), \overline{w}(r))$$

$$\begin{align*}
\tilde{w} &= [\underline{w}(r), \overline{w}(r)] = [(a - (1 - r)\alpha)c + a(c - (1 - r)\gamma) - ac, \\
& \quad (b + (1 - r)\beta)d + b(d + (1 - r)\delta) - bd], \\
& = [ac - (1 - r)(a\gamma + c\alpha), bd + (1 - r)(b\delta + d\beta)].
\end{align*}$$

hence we have:

$$\tilde{w} = \tilde{u} \otimes \tilde{v} = ac, bd, a\gamma + c\alpha, b\delta + d\beta.$$
if \( \tilde{u} \) positive fuzzy number dan \( \tilde{v} \) negative fuzzy number:

\[
\begin{align*}
\tilde{w}(r) &= \tilde{u}(r)\tilde{v}(1) + \tilde{u}(1)\tilde{v}(r) - \tilde{u}(1)\tilde{v}(1), \\
\tilde{\omega}(r) &= \tilde{u}(r)\tilde{\omega}(1) + \tilde{u}(1)\tilde{\omega}(r) - \tilde{u}(1)\tilde{\omega}(1).
\end{align*}
\]  

Based on the formula (8) it is obtained: 
\[
\tilde{w} = [\tilde{w}(r), \tilde{\omega}(r)] = [(b + (1-r)\beta)c + b(c - (1-r)\gamma) - ac, (a - (1-r)\alpha)d + a(d + (1-r)\delta) - ad]
\]  

\[=[bc - (1-r)(b\gamma - c\beta), ad + (1-r)(a\delta + d\alpha)]\]

hence we have:

\[
\tilde{w} = \tilde{u} \otimes \tilde{v} = bc, ad, b\gamma - c\beta, a\delta - d\alpha
\]

if \( \tilde{u} \) negative fuzzy number and \( \tilde{v} \) are positive fuzzy number:

\[
\begin{align*}
\tilde{w}(r) &= \tilde{u}(r)\tilde{v}(1) + \tilde{u}(1)\tilde{v}(r) - \tilde{u}(1)\tilde{v}(1), \\
\tilde{\omega}(r) &= \tilde{u}(r)\tilde{\omega}(1) + \tilde{u}(1)\tilde{\omega}(r) - \tilde{u}(1)\tilde{\omega}(1)
\end{align*}
\]  

Based on the formula (9) it is obtained: 
\[
\tilde{w} = [\tilde{w}(r), \tilde{\omega}(r)] = [(a - (1-r)\alpha)d + a(d - (1-r)\delta) - ad, (b + (1-r)\beta)c + b(c - (1-r)\gamma) - bc]
\]  

\[=[ad - (1-r)(-a\gamma + d\alpha), bd + (1-r)(-b\gamma + c\beta)]\]

hence we have:

\[
\tilde{w} = \tilde{u} \otimes \tilde{v} = ad, bc, d\alpha - a\delta, c\beta - b\gamma.
\]

if \( \tilde{u} \) negative fuzzy number and \( \tilde{v} \) negative fuzzy number:

\[
\begin{align*}
\tilde{w}(r) &= \tilde{u}(r)\tilde{v}(1) + \tilde{u}(1)\tilde{v}(r) - \tilde{u}(1)\tilde{v}(1), \\
\tilde{\omega}(r) &= \tilde{u}(r)\tilde{\omega}(1) + \tilde{u}(1)\tilde{\omega}(r) - \tilde{u}(1)\tilde{\omega}(1)
\end{align*}
\]  

Based on the formula (10) it is obtained: 
\[
\tilde{w} = [\tilde{w}(r), \tilde{\omega}(r)] = [(b + (1-r)\beta)d + b(d + (1-r)\gamma) - bd, (a - (1-r)\alpha)c + a(c - (1-r)\gamma) - ac]
\]  

\[=[bd - (1-r)(-b\delta - d\beta), ac + (1-r)(-a\gamma - c\alpha)]\]

hence we have:

\[
\tilde{w} = \tilde{u} \otimes \tilde{v} = bd, ac, -(b\delta + d\beta), -(a\gamma + c\alpha)
\]
4. SOLVING DFFLS

One method that can be used to determine the solution of a dual fully fuzzy linear equation system is the LU factorization method of the matrix coefficient value. In this method to determine the solution of the system of equations then what will be operated is the value of the matrix coefficient. The steps are as follows:

1. the dual fully fuzzy linear equation system is given as follow:

\[
\tilde{a}_{11}\tilde{x}_1 \oplus \tilde{a}_{12}\tilde{x}_2 \oplus \cdots \oplus \tilde{a}_{1n}\tilde{x}_n = \tilde{b}_{11}\tilde{x}_1 \oplus \tilde{b}_{12}\tilde{x}_2 \oplus \cdots \oplus \tilde{b}_{1n}\tilde{x}_n \oplus \tilde{d}_1,
\]

\[
\tilde{a}_{21}\tilde{x}_1 \oplus \tilde{a}_{22}\tilde{x}_2 \oplus \cdots \oplus \tilde{a}_{2n}\tilde{x}_n = \tilde{b}_{21}\tilde{x}_1 \oplus \tilde{b}_{22}\tilde{x}_2 \oplus \cdots \oplus \tilde{b}_{2n}\tilde{x}_n \oplus \tilde{d}_2.
\]

2. Changing system equations linear fully fuzzy dual at 1 point into form; matrix \(A\) and \(C\):

\[
A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad C = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}
\]

3. we obtain LU decomposition for matrix \(A\) and \(C\), as follows:

\[
A = \begin{bmatrix} 1 & 0 \\ L_{21} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix}
\]

\[
C = \begin{bmatrix} 1 & 0 \\ L_{21} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix}
\]

4. After getting the LU matrix from the matrix \(A\) and \(C\), then we can determine the matrix \(U_2, U_3, U_4\), that is from the inverse multiplication matrix \(L\) \((L^{-1})\) with entries in the DFFLS.

5. The last step is to determine the approximate value for a dual fully fuzzy linear equation system solution, \(x, y, z, w\), is as follows:

\[
x = (L_1U_1 - L_1^0U_1^0)^{-1}f_1,
\]

\[
y = (L_1U_2 - L_1^0U_2^0)^{-1}f_2,
\]

\[
z = (L_1U_1 - L_1^0U_3^0)^{-1}[f_3 - (L_1U_1 - L_1^0U_3^0)x],
\]

\[
w = (L_1U_2 - L_1^0U_3^0)^{-1}[f_4 - (L_1U_1 - L_1^0U_3^0)y].
\]

Computational Example In this section there will be two examples of dual fully fuzzy linear equation system \(\tilde{A} \otimes \tilde{x} \oplus \tilde{c} = \tilde{B} \otimes \tilde{x} \oplus \tilde{d}\) and \(\tilde{A} \otimes \tilde{x} = \tilde{B} \otimes \tilde{x} \oplus \tilde{d}\).
Example 1.1 solve the following dual fully fuzzy linear system $\tilde{A} \otimes \tilde{x} \oplus \tilde{c} = \tilde{B} \otimes \tilde{x} \oplus \tilde{d}$:

$$(3, 4, 2, 2)\tilde{x}_1 \oplus (2, 5, 1, 1)\tilde{x}_2 \oplus (3, 5, 1, 3) = (1, 3, 1, 1)\tilde{x}_1 \oplus (1, 2, 1, 1)\tilde{x}_2 \oplus (7, 20, 8, 13),$$

$$(2, 3, 1, 1)\tilde{x}_1 \oplus (1, 4, 1, 1)\tilde{x}_2 \oplus (2, 3, 3, 2) = (1, 2, 1, 1)\tilde{x}_1 \oplus (2, 3, 2, 1)\tilde{x}_2 \oplus (1, 10, 1, 5).$$

Solution:

1. change system of equations linear fully fuzzy dual in point above to form matrix $A$ and $C$:

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix},$$

2. determine the LU value of the matrix $A$ and $C$,

$$A = \begin{bmatrix} 1 & 0 \\ 2/3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.$$  

$$A = \begin{bmatrix} 1 & 0 \\ 3/2 & -1/3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$$  

3. After getting the LU matrix from the $A$ and $C$, we can then determine the matrix $U_2, U_3, U_4$, that is from the inverse multiplication matrix $L(L^{-1})$ the entries in the DFFLS are as follows:

determine $L_1^{-1}$ and $(L^o)_1^{-1}$.

$$L_1^{-1} = \begin{bmatrix} 1 & 0 \\ -2/3 & 1 \end{bmatrix}, \quad (L^o)_1^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}.$$  

then obtained:

$U_2 = L_1^{-1} \cdot M$

$$U_2 = \begin{bmatrix} 1 & 0 \\ -2/3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 5/3 & 2/3 \end{bmatrix}.$$  

$U_3 = L_1^{-1} \cdot N$

$$U_3 = \begin{bmatrix} 1 & 0 \\ -2/3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1/3 & 1/3 \end{bmatrix}.$$
\[ U_4 = L_1^{-1} \cdot R \]

\[ U_4 = \begin{bmatrix} 1 & 0 \\ -2/3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1/3 & 1/3 \end{bmatrix} \]

then for \( U_2^{\text{circ}}, U_3^{\text{circ}}, U_4^{\text{circ}} \) is as follows:

\[ U_2^o = L_1^{-1} \cdot M^o \]

\[ U_2^o = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix} \]

\[ U_3^o = L_1^{-1} \cdot N^o \]

\[ U_3^o = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \]

\[ U_4^o = L_1^{-1} \cdot R^o \]

\[ U_2^o = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \]

after getting the value of \( U \) we will determine the value of \( \tilde{F}_{u1} \) and \( \tilde{F}_{v1} \) which is:

\[ \tilde{f}_u = (\tilde{f}_{u11}, \tilde{f}_{u12}, \tilde{f}_{u13}, \tilde{f}_{u14} - \tilde{f}_{u21}, \tilde{f}_{u22}, \tilde{f}_{u23}, \tilde{f}_{u24}) \]

\[ \tilde{f}_u = (7, 20, 8, 13) \odot (3, 5, 1, 3) \]

\[ \tilde{f}_u = (7, 20, 8, 13) \odot (5, 3, -3, -1) \]

and

\[ \tilde{f}_v = (\tilde{f}_{v11}, \tilde{f}_{v12}, \tilde{f}_{v13}, \tilde{f}_{v14} - \tilde{f}_{v21}, \tilde{f}_{v22}, \tilde{f}_{v23}, \tilde{f}_{v24}) \]

\[ \tilde{f}_v = (1, 10, 1, 5) \odot (2, 3, 3, 2) \]

\[ \tilde{f}_v = (1, 10, 1, 5) \odot (3, 2, -2, -3) \]

so that

\[ \tilde{f} = \begin{bmatrix} 4, 15, 7, 10 \\ -1, 7, -2, 3 \end{bmatrix} \]

4. the last step is to determine the approximate value for the dual fully fuzzy linear equation system solution, namely, \( x, y, z, w \), as follows:

\[ x = (L_1 U_1 - L_1^o U_1^o)^{-1} f_1, \]

\[ y = (L_1 U_2 - L_1^o U_2^o)^{-1} f_2, \]

\[ z = (L_1 U_3 - L_1^o U_3^o)^{-1} [f_3 - (L_1 U_3 - L_1^o U_3^o)x], \]

\[ w = (L_1 U_2 - L_1^o U_2^o)^{-1} [f_4 - (L_1 U_3 - L_1^o U_3^o)y]. \]
So that it is obtained \( x = (L_1 U_1 - L_1^o U_1^o)^{-1} f_1 \), or
\[
x = \begin{bmatrix}
  1/3 & 1/3 \\
  1/3 & -2/3
\end{bmatrix}
\begin{bmatrix}
  4 \\
  -1
\end{bmatrix}
= \begin{bmatrix}
  1 \\
  2
\end{bmatrix}
\]

\( y = (L_1 U_2 - L_1^o U_2^o)^{-1} f_2 \), or
\[
y = \begin{bmatrix}
  1/3 & 1/3 \\
  1/3 & -2/3
\end{bmatrix}
\begin{bmatrix}
  6 \\
  0
\end{bmatrix}
= \begin{bmatrix}
  2 \\
  2
\end{bmatrix}
\]

\( z = (L_1 U_1 - L_1^o U_1^o)^{-1} [f_3 - (L_1 U_3 - L_1^o U_3^o) x] \), or
\[
z = \begin{bmatrix}
  1/3 & 1/3 \\
  1/3 & -2/3
\end{bmatrix}
\begin{bmatrix}
  6 \\
  0
\end{bmatrix}
= \begin{bmatrix}
  2 \\
  2
\end{bmatrix}
\]

\( w = (L_1 U_2 - L_1^o U_2^o)^{-1} [f_4 - (L_1 U_3 - L_1^o U_3^o) y] \), or
\[
w = \begin{bmatrix}
  -1/2 & 3/2 \\
  1/2 & -1/2
\end{bmatrix}
\begin{bmatrix}
  7 \\
  3
\end{bmatrix}
= \begin{bmatrix}
  1 \\
  2
\end{bmatrix}.
\]

So a solution is obtained \( \tilde{x}_1 = (x_1, y_1, z_1, w_1) = (2, -5, 1, -12), \)
\( \tilde{x}_2 = (x_2, y_2, z_2, w_2) = (1, 6, 0.5, 4) \).
5. CONCLUDING REMARKS

In this paper, we have discussed the solution of a dual fully fuzzy linear equation system in the form of \( \tilde{A} \tilde{x} \oplus \tilde{b} = \tilde{B} \otimes \tilde{x} \oplus \tilde{d} \) is then transformed into the form \( A \otimes \tilde{x} = B \otimes \tilde{x} \oplus \tilde{f} \). The solution method uses LU factorization from the value of the matrix coefficient with the fuzzy trapezoid number. In the example, the completed fully fuzzy linear equations system is solved by using positive trapezoid fuzzy number.

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