International Journal of Architecture and Urbanism Vol. 08, No. 03 (2024) 448 – 462

International Journal of Architecture and Urbanism

Journal homepage: https://talenta.usu.ac.id/ijau

Research on Determination of Bearing Capacity of Existing Reinforced Concrete Beam Considering Probability Distribution Characteristics

Kang Il Chan*[1], Ri Chol Myong[1]

[1] *Pyongyang University of Architecture, Pyongyang, DPRK*

*Corresponding Author: shypinguo202132@126.com

ARTICLE INFO

Article history:
Received 12-6-2024
Revised 11-11-2024
Accepted 14-11-2024
Available online 30-11-2024

E-ISSN: 2622-1640
P-ISSN: 2622-0008

How to cite:
Chang K. I., Myong R. C. Kamil E. M. Research on Determination of Bearing Capacity of Existing Reinforced Concrete Beam Considering Probability Distribution Characteristics. International Journal of Architecture and Urbanism. 2024. 8(3):448-462.

http://doi.org/10.32734/ijau.v8i3.16799

ABSTRACT

The reliability of existing buildings is assessed in terms of safety, applicability and durability, and the main aspect of safety related with safety assessment is the assessment of bearing capacity. However, the bearing capacity of existing building structures decreases with increasing the length of operational time, and its accurate assessment can ensure the safety of existing buildings, while extending the life span of buildings and avoiding unnecessary repair and strengthening measures. This paper determined the bearing capacity of existing reinforced concrete beams by considering the probability distribution characteristics of technical assessment data for existing buildings, entities which are objectively existing, unlike designed structures.

Keywords: bearing, probability, reinforced concrete beam,

1. Introduction

In general, buildings are designed according to the period of design criterion and managed according to their reliability, and are removed when they cannot improve reliability through various management measures including repair and strengthening [1]. Currently, there are many buildings and structures that are close to or exceeding the design criteria in the world, and as time goes on, they gradually disappear and they are subjected to various factors including natural disasters, and they are destructed or damaged [2]. Therefore, the building management process of reliability analysis at the new establishment stage should normally be assessed for reliability, which is composed of safety, applicability and durability to maintain the acceptable reliability of existing building structures as expected in the design [3]. However, the reliability assessment of existing structures has the following characteristics different from the reliability analysis of new structures.

Buildings are traditionally designed according to the prevailing design criteria at the time of their construction. These criteria account for anticipated load capacities, environmental conditions, and intended usage, emphasizing factors like safety, applicability, and durability. However, as buildings age, they frequently exceed these initial design parameters, confronting additional external factors such as natural wear, environmental stresses, and sporadic extreme events, including natural disasters. Consequently, maintaining acceptable levels of structural reliability in aging buildings necessitates not only regular monitoring but also periodic interventions, such as repair and strengthening, whenever feasible [7][8]. Reliability assessment plays a crucial role in the management of existing structures and differs significantly from the reliability evaluation of new constructions. According to ISO 13822:2003 [4], the reliability of a new structure is defined as the probability that it will fulfill its intended function over a specified time under predefined conditions. For existing buildings, however, this definition requires adaptation to account for changes in the structure’s operational and environmental context, remaining service life, and actual performance history [9][10][11]. The primary dimensions of reliability—safety, applicability, and durability—are influenced by factors unique to the service history and condition of existing structures. Safety, commonly measured through bearing capacity, typically diminishes over time as material degradation and cumulative damage compromise structural integrity. Applicability and durability are similarly impacted by phenomena such as material fatigue, corrosion in reinforced concrete elements, and shifts in the functional requirements of the building [12][13][14]. For existing buildings, factors like load history, environmental exposure, and operational changes (e.g., shifts in occupancy or increased loading demands) are critical to reliability assessment. Unlike newly designed structures, which only consider "normal conditions" anticipated within the design period, existing structures must account for “verified load” conditions-loads and stresses they have already experienced throughout their lifespan. Assessing an existing building's reliability thus requires empirical data from technical evaluations, such as material strength tests and load-bearing capacity assessments, which introduce complex variables into the analysis [15][16]. Modern approaches to reliability evaluation incorporate probabilistic models to account for the inherent variability in material properties and structural responses over time, as demonstrated in studies on damage detection and reliability evaluation, including the work of Yao and Natke [6]. Probability-based models for assessing bearing capacity, for instance, integrate statistical variables that offer data-driven insights into the current structural state. For flexural members, probabilistic methods are often applied to analyze cracking behavior and load-bearing capacity, as explored in ISO 2394:1998 and research on crack control in reinforced concrete [5]. The management and assessment of aging building structures thus require an evolution from static design criteria to a dynamic reliability framework, incorporating probabilistic models and periodic technical evaluations. Such an approach enables a more accurate assessment of bearing capacity, optimizes the lifecycle of existing structures, and ensures that safety, durability, and functionality standards are effectively sustained or enhanced.

Difference of reliability definition elements

The reliability of a structure to be newly constructed is a probability criteria of reliability, whose exact meaning is the probability that the structure performs a predetermined function within a specified time, under specified conditions. Here, the elements defining reliability, ‘time’, ‘condition’, ‘function’ is somewhat different from the existing structure. The "specified time" ($t_0, t_0 + T_s$, t_0: the period of service of existing structure, T_s: period of loading criteria) is not the period of design criteria of the newly constructed structure, but the remaining length of life time determined by the current state of the structure and the environment of service. The different time intervals for analyzing and assessing the reliability of the new and existing structures, the bearing capacity and load combination for reliability determination are also different. In the new structure, “specified conditions” refer to “three normal conditions”, while “specified conditions” of the existing structure refers to both the operation and maintenance conditions. The operation conditions described here include actions that are not considered when designing, but maintenance conditions include maintenance conditions for the structure and control of the surrounding environment that are carried out to meet future operational objectives or to increase the reliability of the structure. The future 'intended' function of the existing structure is generally referred to as the function set up in the new design, but may be changed by the activities of people, such as the change of the purpose of some buildings, the reconstruction and expansion. Since the meaning of these three elements in the definition of reliability of an existing structure is different from that of a new structure, the definition of reliability of an existing structure has some difference from that of a new structure, and therefore attention should be paid to this feature when assessing an existing structure.

Variation of load and bearing capacity affected by existing structures

When a structure is used for a certain period or is subjected to a certain amount of damage, various degrees of damage will occur, and these damage will necessarily affect the load effect and the reliability of the structure during its future operation.

Compared with the new structure, the maximum feature of the existing structural load is the "verified load". In other words, the structure has already been subjected to some load and its combination. The response caused by damage to structures such as long-term loading, deterioration of structural material performance, corrosion of reinforced concrete, structural cross-section changes and crack initiation, etc. is different from that of new structures. Taking an objective look at the bearing capacity of an existing structure, this is a definite quantity, but it must be analyzed by technical assessment data because the understanding of the bearing capacity of the structure is not comprehensive due to the influence of the current measurement instruments and some subjective factors.

Problem of Maximum Load Value

The maximum load value is directly related to the design criteria period of the structure, and the criteria period of the existing structure (the continuously used criteria period) is determined comprehensively by the production requirements and the technical state of the structure, etc., and is generally less than the design criteria period of the structure. Hence, the maximum load value of the existing structure is less than the design load and should be determined again by the length of the continuous service life.

As shown above, to accurately assess the bearing capacity of existing reinforced flexural members, the technical assessment data and probability characteristics of existing structures should be used in a rational way. Therefor this paper presents an assessment approach to evaluating the bearing capacity in existing reinforced concrete flexural members, which integrates the measured results of the material properties directly related to the bearing capacity, and the measured values of the material properties indirectly related to the bearing capacity, together with the measured results of the geometrical variables.

2. Method

This study evaluates the bearing capacity of existing reinforced concrete beams by incorporating probabilistic characteristics into the assessment process. A comparative analysis was conducted between the traditional approach—focusing solely on the correlation between crack width and bearing capacity—and a new method that considers the probability distribution of primary variables affecting bearing capacity.

Data collection and field measurements were conducted to gather empirical data on the dimensions, material properties, and structural conditions of reinforced concrete beams. Key measurements included the actual beam width (reb) and height (reh), as well as the concrete compressive strength. In addition, the maximum crack width (amax) was recorded for various beam samples under typical loading conditions.

To assess bearing capacity, a probabilistic model was constructed based on the lognormal distribution of key variables affecting structural strength. The bending bearing capacity M was calculated using an adapted formula from the "Concrete Structure Design Criterion," incorporating variables such as beam dimensions, material strength, and reinforcement configuration. The model applies Taylor series expansion at the mean point to obtain the mean and standard deviation, providing statistical variables for both the traditional and the new method.

This analysis employs statistical coefficients (e.g., A1 to A6) to account for the random variability in factors affecting bearing capacity, such as concrete strength (fc), beam dimensions, and reinforcement properties. Each coefficient was calculated according to standardized formulas to meet the criteria outlined in structural design codes.

For reinforced concrete beams with known crack widths, the probability distribution function for the bearing capacity M was revised using the measured values of crack width, geometry, and material properties. The correction factor KM was applied to align the calculated bearing capacity with observed field data, ensuring accuracy in capacity estimations for both assessment methods.

The final step involved comparing results from the traditional and new methods under varying crack widths (0.16 mm, 0.20 mm, and 0.24 mm). Each method's results were analyzed to assess differences in calculated bearing capacity and the coefficient of variation. These findings aim to demonstrate the enhanced reliability and robustness of the new method in predicting the loadbearing capacity of existing beams.

3. Result and Discussion

Characteristics of the bearing capacity and probability distribution with maximum crack width of the existing reinforced concrete beams.

Probability distribution characteristics of bearing capacity

The actual bending capacity of reinforced concrete beams is affected by various random factors, and the uncertainty coefficient K_M of the calculation schema should be applied, since there is always a difference between the theoretical and actual values. Taking the diameter d, $A_s = \frac{\pi}{4} nd^2$ (n is the number of longitudinal bars in the tensile zone) by the formula for the calculation of the bearing capacity of reinforced concrete single-bar rectangular flexural members, as presented in the "Concrete Structure Design Criterion", the actual bending bearing capacity of the structural members is given as follows.

$$M = K_M \frac{n\pi d^2}{4} h_0 f_s \left(1 - \frac{n\pi d^2 f_s}{8\alpha_1 f_c b h_0}\right) \tag{1}$$

In this equation, K_M, d, h_0, b, f_s, f_c is basic random variables that are independent each other, and the meaning of each symbol is the same as ones presented in the "concrete structure design criterion". Expanding Eq. (1) to the Taylor series at the mean point and taking the linear term, we obtain the result of statistical analysis of the probability characteristics as follows (the mean value and standard deviation).

$$M \approx g\left(\mu_{K_M}, \mu_{h_0}, \mu_d, \mu_b, \mu_{f_c}, \mu_{f_s}\right) + A_1(h_0 - \mu_{h_0}) + A_2(d - \mu_d) +$$

$$A_3(b - \mu_b) + A_4(f_c - \mu_{f_c}) + A_5(f_s - \mu_{f_s}) + A_6(K_M - \mu_{K_M})$$

$$\mu_M^/ = g\left(\mu_{K_M}, \mu_{h_0}, \mu_d, \mu_b, \mu_{f_c}, \mu_{f_s}\right) \tag{2}$$

$$\sigma_M^/ = \sqrt{A_1^2\delta_{h_0}^2\mu_{h_0}^2 + A_2^2\delta_d^2\mu_d^2 + A_3^2\delta_b^2\mu_b^2 + A_4^2\delta_{f_c}^2\mu_{f_c}^2 + A_5^2\delta_{f_s}^2\mu_{f_s}^2 + A_6^2\delta_{K_M}^2\mu_{K_M}^2} \tag{3}$$

$$\delta_M^/ = \frac{\sigma_M^/}{\mu_M^/} \tag{4}$$

Where μ_{Xi} - Mean value of random variable

$\mu_M^/$ - Mean value of member bearing capacity.

$\sigma_M^/$ _ Standard deviation of member bearing capacity

$\delta_M^/$ - Variance coefficient of member bearing capacity

$A_1 \sim A_6$ - The main variable, it is calculated by Eq. $A_i = \frac{\partial f(X_i)}{\partial X_i}\Big|_m$

The coefficient equation is given in Table 1. The meaning of the other symbols is the same as before.

Table 1. Calculation of the coefficients of the main variables

coefficient	Calculation formula
A_1	$0.25n\pi K_M d^2 f_s\big\|_m$
A_2	$0.5n\pi d h_0 f_s K_M - n^2\pi^2 K_M f_s^2 d^3 / 8bf_c\big\|_m$
A_3	$n^2\pi^2 K_M d^4 f_s^2 / 32b^2 f_c\big\|_m$
A_4	$n^2\pi^2 K_M d^4 f_s^2 / 32bf_c^2\big\|_m$
A_5	$0.25n\pi K_M d^2 h_0 - n^2\pi^2 K_M d^4 f_s / 16bf_c\big\|_m$
A_6	$0.25n\pi d^2 h_0 f_s (1 - n\pi d^2 f_s / 8 f_c b h_0)\big\|_m$

Note: The subscript m represents the mean value of each basic random variable.

It can be seen that the bearing capacity of flexural member is approximately followed by the regular logarithmic distribution by the central limit theorem of probability theory.

Probability distribution characteristics of maximum crack width

Since the magnitude of the maximum crack width that can actually occur in a structural member is influenced by various random factors, there always exists a difference between the theoretical and actual values of the maximum crack width of the member, the uncertainty coefficient K_a of the calculation schema should be applied. By calculating the maximum crack width a_{crc} of flexural members, calculated by considering the influence of long-term behavior , according to the combination of the loading criteria presented in the "Reinforced Concrete Structure Design Criterion", the formula for the maximum crack width a_{crc} that can actually occur in structural members can be obtained as follows :

$$a_{crc} = \alpha_c \psi_s \frac{\sigma_s}{E_s} \cdot \left(2.65c + 0.1\frac{d}{\mu_{ef}}\right)\eta_0 \tag{6}$$

Where

α_c - coefficient related to the load bearing characteristics of the member.

For bending and eccentric compression, α_c = 2.1.

For eccentric tension, α_c = 2.4

For central tension, α_c = 2.7

ψ_s - coefficient considering the inequality of tension reinforcement stress

$$\psi_s = 1.25 - \varphi_\ell \frac{M_{crc}}{M_n}$$

η_0 - Formation factor of reinforcement when circular reinforcement.

For round bar $\eta_0 = 1.0$

For deformed bar $\eta_0 = 0.7$

σ_s _ Stress of tension reinforcement , MPa

For flexural member

$$\sigma_s = \frac{M_n}{A_s \cdot 0.87 h_0}$$

c- distance from the bottom of the tension reinforcement in the bottom row to the bottom of the tension zone, mm

d _The diameter of the steel section, mm, and diameter of different bars are taken as follows.

$$d = \frac{n_1 d_1{}^2 + n_2 d_2{}^2 + \cdots + n_i d_i{}^2}{n_1 d_1 + n_2 d_2 + \cdots + n_i d_i}$$

μ_{ef} -Effective bar arrangement ratio

$$\mu_{ef} = \frac{A_s}{A_{c,ef}}$$

$$A_{c,ef} = 0.5bh + (b_f - b)h_f$$

Considering that the member mentioned here is a single-bar rectangular reinforced concrete member subjected to bending d, the reinforced bar is a round steel bar and bar diameter is the same, if uncertainty factor of the crack model is applied, Eq. (6) can be expressed as follows:

$$a_{crc} = 2.1\mathrm{K}_a [\frac{5.75(M_{Gn} + M_{Qn}) - 1.343 f_{ctn} bh^2 - 17.25 n\pi d^2 f_{ctn} h}{E_s h_0 n\pi d^2}] \cdot (\frac{0.2bh + 2.65 n\pi dh - 2.65 n\pi dh_0 - 1.325 n\pi d^2}{n\pi d}) \quad (7)$$

Where K_a - Uncertainty Coefficient of Computational Schema

f_{ctn} - Baseline tensile strength of concrete.

b – width of beam section

h – height of beam section

G_n – Baseline Fixed Load

Q_n – Baseline temporary load

n- Number of re-bars

h_0 - Effective section height

E_s - modulus of elasticity of reinforcement

d- diameter of reinforcement

In Eq. (7), $K_a, E_s, h_0, d, M_{Gn}, M_{Qn}, f_{ctn}, b, h$ is a nine basic random variable that are mutually independent and the meaning of each symbol is the same as mentioned above.

If the statistical results (mean, standard deviation) of the probability characteristics are obtained by expanding Eq. (7) to the Taylor series at the mean point μ_{Xi} and taking only the linear term we will get the following equations

$$a \approx f\left(\mu_{K_a}, \mu_{h_0}, \mu_d, \mu_b, \mu_h, \mu_{f_{ctn}}, \mu_{M_{Gn}}, \mu_{M_{Qn}}, \mu_{E_s}\right) + B_1(h_0 - \mu_{h_0}) +$$
$$B_2(d - \mu_d) + B_3(b - \mu_b) + B_4\left(f_{ctn} - \mu_{f_{ctn}}\right) + B_5(h - \mu_h) + B_6\left(E_s - \mu_{E_s}\right) +$$
$$B_7\left(M_{Gn} - \mu_{M_{Gn}}\right) + B_8\left(M_{Qn} - \mu_{M_{Qn}}\right) + B_9\left(K_a - \mu_{K_a}\right) \tag{8}$$

$$\mu_a^{/} = f\left(\mu_{K_a}, \mu_{h_0}, \mu_d, \mu_b, \mu_{f_{ctn}}, \mu_h, \mu_{E_s}, \mu_{M_{Gn}}, \mu_{M_{Qn}}\right) \tag{9}$$

$$\sigma_a^{/} = (B_1^2\delta_{h_0}^2\mu_{h_0}^2 + B_2^2\delta_d^2\mu_d^2 + B_3^2\delta_b^2\mu_b^2 + B_3^2\delta_b^2\mu_b^2 + B_4^2\delta_{f_{ctn}}^2\mu_{f_{ctn}}^2 + B_5^2\delta_h^2\mu_h^2 +$$
$$B_6^2\delta_{h_0}^2\mu_{h_0}^2 + B_7^2\delta_{M_{Gn}}^2\mu_{M_{Gn}}^2 + B_8^2\delta_{M_{Qn}}^2\mu_{M_{Qn}}^2 + B_9^2\delta_{K_a}^2\mu_{K_a}^2)^{1/2} \tag{10}$$

$$\delta_a^{/} = \frac{\sigma_a^{/}}{\mu_a^{/}} \tag{11}$$

where

μ_{Xi} - Mean value of random variable

$\mu_a^{/}$ - Mean value of maximum crack width.

$\sigma_a^{/}$ - standard deviation of maximum crack width

$\delta_a^{/}$ - coefficient of variation of maximum crack width

$B_1 \sim B_9$ - The main variable is calculated by the equation $B_i = \frac{\partial g(X_i)}{\partial X_i}|_m$, and the calculated results are shown in Table 2.

Table 2. Coefficient calculation formula of the main variables

coefficient	Calculation formula
B_1 (h₀)	$-\frac{5.565K_\omega}{E_s n h_0 \pi d^2}\left[5.75(M_G+M_Q)-1.343bf_t h^2-17.25n\pi f_t hd^2\right]-\frac{2.1K_\omega}{E_s n^2 {h_0}^2 \pi^2 d^3}\left[\frac{bh}{5}-1.325n\pi d^3+2.65n\pi d(h-h_0)\right]\left[\begin{matrix}5.75(M_G+M_Q)-1.343bf_t h^2- \\ 17.25n\pi f_t hd^2\end{matrix}\right]\Big\|_m$
B_2 (d)	$-\frac{6.3K_\omega}{E_s n^2 h_0 \pi^2 d^4}\left[\frac{bh}{5}-1.325n\pi d^2+2.65n\pi d(h-h_0)\right]\left[5.7(M_G+M_Q)-1.34bf_t h^2-17.2n\pi f_t hd^2\right]$ $-\frac{2.1K_\omega}{E_s n^2 h_0 \pi^2 d^3}\left[2.65n\pi d-2.65n\pi(h-h_0)\right]\left[5.7(M_G+M_Q)-1.34bf_t h^2-17.2n\pi f_t hd^2\right]$ $\frac{72.45K_\omega f_t h}{E_s n h_0 \pi d^2}\left[\frac{bh}{5}-1.325n\pi d^2+2.65n\pi d(h-h_0)\right]\Big\|_m$
B_3 (b)	$\frac{0.42K_\omega h}{E_s n^2 h_0 \pi^2 d^3}\left[5.75(M_G+M_Q)-1.343bf_t h^2-17.25n\pi f_t hd^2\right]-\frac{28.2K_\omega f_t h^2}{E_s n^2 h_0 \pi^2 d^3}\left[\frac{bh}{5}-1.325n\pi d^3+2.65n\pi d(h-h_0)\right]\Big\|_m$
B_4 (f)	$-\frac{2.1K_\omega}{E_s n^2 h_0 \pi^2 d^3}\left[\frac{bh}{5}-1.325n\pi d^2+2.65n\pi d(h-h_0)\right]\left[1.343bh^2+17.25n\pi hd^2\right]\Big\|_m$
B_5 (h)	$\frac{2.1K_\omega}{n^2\pi^2 d^3 E_s h_0}\left\{\left[\frac{bh}{5}+2.65n\pi d\right]\left[5.75(M_G+M_Q)-1.343bf_t h^2-17.25n\pi f_t hd^2\right]-\left[17.25\pi f_t nd^2+2.68bf_t h\right]\left[\frac{bh}{5}-1.325n\pi d^2+2.65n\pi d(h-h_0)\right]\right\}\Big\|_m$
B_6 (E)	$-\frac{2.1K_\omega}{n^2\pi^2 d^3 {E_s}^2 h_0}\left[\frac{bh}{5}-1.325n\pi d^2+2.65n\pi d(h-h_0)\right]\left[5.75(M_G+M_Q)-\right.$ $\left.-1.343bf_t h^2-17.25n\pi f_t hd^2\right]_m$
B_7, B_8 (M)	$\frac{12.075K_\omega}{E_s n^2 h_0 \pi^2 d^3}\left[\frac{bh}{5}-1.325n\pi d^2+2.65n\pi d(h-h_0)\right]\Big\|_m$
B_9 (k)	$\frac{2.1}{E_s n^2 h_0 \pi^2 d^3}\left[\frac{bh}{5}-1.325n\pi d^2+2.65n\pi d(h-h_0)\right]\left[5.75(M_G+M_Q)-1.343bf_t h^2\right.$ $\left.-17.25n\pi f_t hd^2\right]_m$

Note: In the table above, the subscript m gives the average value of each random variable.

By the central limit theorem of probability theory, it can be seen that the maximum crack width approximately follows a lognormal distribution. That is:

$$a \sim Ln\left[\mu_{\ln a}{}', \left(\sigma_{\ln a}{}'\right)^2\right] \tag{12}$$

Correlation between maximum crack width and bearing capacity

Both the bearing capacity and the maximum crack width are universal random variables, and there is strong correlation between them through common geometric and material performance variables, and the correlation coefficient between them is

$$\rho'_{M\omega} = \frac{1}{\delta'_M \delta'_\omega}\left(g_1\delta^2_{K\omega} + g_2\delta^2_d + g_3\delta^2_b + g_4\rho_{f_s f_c}\delta_{f_t}\delta_{f_c}\right) \tag{13}$$

g_1, g_2, g_3, g_4 the normal variables associated with the mean value of each major variable and they are calculated with Eq. (14) ~ (17).

$$g_1 = \frac{B_1 A_1}{B_9 A_6}\mu^2_{h_0} \tag{14}$$

$$g_2 = \frac{B_2 A_2}{B_9 A_6}\mu^2_d \tag{15}$$

$$g_3 = \frac{B_3 A_3}{B_9 A_6}\mu^2_b \tag{16}$$

$$g_4 = \frac{B_4 A_4}{B_9 A_6}\mu_{f_t}\mu_{f_c} \tag{17}$$

Influence of technical evaluation results on the probability distribution characteristics of maximum crack width and bearing capacity

Through technical assessment or field measurements of existing reinforced members, the geometrical dimension of the member, the concrete compressive strength and etc. can be obtained directly and applied according to data processing principles. In other words, if a measured value can accurately reflect the actual value, the reference value of this variable must be a measured value, the corresponding mean value be a measured value, and the coefficient of variation be zero.

Effect of measurement results of geometrical parameters on the probability distribution characteristics of maximum crack width and bearing capacity

By measuring the flexural members of reinforced concrete on the field, the actual width b_{re} and the actual height h_{re} of the members can be accurately obtained.

The following values are taken then.

$$\mu_b = b_{\text{re}} \quad \delta_b = 0 \tag{18}$$

$$\mu_h = h_{\text{re}} \quad \delta_h = 0 \tag{19}$$

Substituting Eq. (18) and (19) into Eq. (2), (3), (9), and (10), respectively, we obtain the following result.

In other words, the statistical variables of flexural member bearing capacity are revised,

$$\mu_M = g\left(\mu_{K_M}, \mu_{h_0}, \mu_d, b_{\text{re}}, \mu_f, \mu_f\right) \tag{20}$$

$$\sigma_M = \sqrt{A_1^2\delta_{h_0}^2\mu_{h_0}^2 + A_2^2\delta_d^2\mu_d^2 + A_4^2\delta_{f_c}^2\mu_{f_c}^2 + A_5^2\delta_{f_s}^2\mu_{f_s}^2 + A_6^2\delta_{K_M}^2\mu_{K_M}^2} \quad (21)$$

If the statistical parameters of maximum crack width in flexural members are revised

$$\mu_a = g\left(\mu_{K_a}, \mu_{h_0}, h_d, b_{\mathrm{re}}, \mu_{f_t}, h_{\mathrm{re}}, \mu_{E_s}, \mu_{M_G}, \mu_{M_Q}\right) \quad (22)$$

$$\sigma_a = (B_1^2\delta_{h_0}^2\mu_{h_0}^2 + B_2^2\delta_d^2\mu_d^2 + B_4^2\delta_{f_t}^2\mu_{f_t}^2 + B_6^2\delta_{E_s}^2\mu_{E_s}^2 + B_7^2\delta_{M_G}^2\mu_{M_G}^2$$

$$+ B_8^2\delta_{M_Q}^2\mu_{M_Q}^2 + B_9^2\delta_{K_a}^2\mu_{K_a}^2)^{1/2} \quad (23)$$

Based on the measured results of the geometrical variables, the revision of the correlation coefficient of the maximum crack width and the bearing capacity of RC flexural members is given by

$$\rho_{Ma} = \frac{1}{\delta_M\delta_a}\left(g_1\delta_{h_0}^2 + g_2\delta_d^2 + g_4\rho_{f_t f_c}\delta_{f_t}\delta_{f_c}\right) \quad (24)$$

The meaning of the symbols of each formula is the same as mentioned earlier.

Effect of measurement results of material strength on the probability distribution characteristics of maximum crack width and bearing capacity

Through the measurements in reinforced concrete flexural members, the actual strength of concrete can be obtained in the same way, taking the following values.

$$\mu_{fc} = f_{cre} \quad \delta_{fc} = 0 \quad (25)$$

Similarly, substituting formula (25) into formula (2) (3) (9) (10), respectively, we obtain the following result.

If the statistical parameters of flexural member bearing capacity is revised

$$\mu_M = g\left(\mu_{K_M}, \mu_{h_0}, \mu_d, \mu_b, f_{cre}, \mu_{f_s}\right) \quad (26)$$

$$\sigma_M = \sqrt{A_1^2\delta_{h_0}^2\mu_{h_0}^2 + A_2^2\delta_d^2\mu_d^2 + A_3^2\delta_b^2\mu_b^2 + A_5^2\delta_{f_s}^2\mu_{f_s}^2 + A_6^2\delta_{K_M}^2\mu_{K_M}^2} \quad (27)$$

The revised value of the correlation coefficient of reinforced flexural member bearing capacity and maximum crack width under the influence of the geometric parameter is as follows:

$$\rho_{Ma} = \frac{1}{\delta_M\delta_\omega}\left(g_1\delta_{h_0}^2 + g_2\delta_d^2 + g_3\delta_b^2\right) \quad (28)$$

Effect of measured geometrical parameters and material strength on the probability distribution characteristics of maximum crack width and bearing capacity

Combining the aforementioned cases, we can see the following equations.

$$\mu_b = b_{\mathrm{re}} \quad \delta_b = 0 \quad (29)$$

$$\mu_h = h_{\mathrm{re}} \quad \delta_h = 0 \quad (30)$$

$$\mu_{fc} = f_{cre} \quad \delta_{fc} = 0 \quad (31)$$

Substituting the above three equations into Eq. (2), (3), (9), and (10) respectively, we can obtain the following result.

If the statistical variable of flexural member bearing capacity is revised

$$\mu_M = g\left(\mu_{K_M}, \mu_{h_0}, \mu_d, b_{\mathrm{re}}, \mathrm{f}_{cre}, \mu_{f_s}\right) \tag{32}$$

$$\sigma_M = \sqrt{A_1^2\delta_{h_0}^2\mu_{h_0}^2 + A_2^2\delta_d^2\mu_d^2 + A_5^2\delta_{f_s}^2\mu_{f_s}^2 + A_6^2\delta_{K_M}^2\mu_{K_M}^2} \tag{33}$$

If the statistical variables of maximum crack width of flexural members are revised

$$\mu_a = g\left(\mu_{K_a}, \mu_{h_0}, \mu_d, b_{\mathrm{re}}, f_t, h_{\mathrm{re}}, \mu_{E_s}, \mu_{M_G}, \mu_{M_Q}\right) \tag{34}$$

$$\sigma_a = (B_1^2\delta_{h_0}^2\mu_{h_0}^2 + B_2^2\delta_d^2\mu_d^2 + B_4^2\delta_{f_t}^2\mu_{f_t}^2 + B_6^2\delta_{E_s}^2\mu_{E_s}^2 + B_7^2\delta_{M_G}^2\mu_{M_G}^2 +$$

$$B_8^2\delta_{M_Q}^2\mu_{M_Q}^2 + B_9^2\delta_{K_\omega}^2\mu_{K_\omega}^2)^{1/2} \tag{35}$$

Under the overall influence of the measured results on the geometrical variables and material strength, the revised value of the correlation coefficient of the flexural members' bearing capacity and the maximum crack width is as follows:

$$\rho_{Ma} = \frac{1}{\delta_M\delta_a}\left(g_1\delta_{h_0}^2 + g_2\delta_d^2\right) \tag{36}$$

Bearing capacity of existing reinforced flexural members.

Statistical variables after revising bearing capacity

From the earlier analysis, it is known that both the bearing capacity and the maximum crack width follow approximately a lognormal distribution.

If the actual maximum crack width value of a flexural member is a_0 that is, $a = a_0$, then the probability distribution function of the bearing capacity M after the revision can be used to reflect the probability characteristics of this flexural member bearing capacity, and then the statistical variables after the revision are as follows :

$$\mu_{M|a=a_0} = \exp[\ln \mu_M - \frac{1}{2}\ln\left(1+\delta_M^2\right) + \rho_{\ln M,\ln a}\sqrt{\ln\left(1+\delta_M^2\right)}\frac{\ln a_0 - \ln \mu_a + \frac{1}{2}\ln\left(1+\delta_a^2\right)}{\sqrt{\ln\left(1+\delta_a^2\right)}}$$

$$+\frac{1}{2}\left(1-\rho_{\ln M,\ln a}^2\right)\ln\left(1+\delta_M^2\right)] \tag{37}$$

$$\sigma_{M|a=a_0} = \sqrt{\exp[\sqrt{\left(1-\rho_{\ln M,\ln a}^2\right)\ln\left(1+\delta_M^2\right)}]-1}\ \exp[\ln \mu_M - \frac{1}{2}\ln\left(1+\delta_M^2\right)$$

$$+\rho_{\ln M,\ln a}\sqrt{\ln\left(1+\delta_M^2\right)}\frac{\ln a_0 - \ln \mu_a + \frac{1}{2}\left(1+\delta_a^2\right)}{\sqrt{\ln\left(1+\delta_a^2\right)}} + \frac{1}{2}\left(1-\rho_{\ln M,\ln a}^2\right)\ln\left(1+\delta_M^2\right)] \tag{38}$$

Where

$\mu_{M\mid a=a_0}, \sigma_{M\mid a=a_0}$ mean value, standard deviation of bearing capacity M after second correction.

μ_M, δ_M mean value, coefficient of variation of bearing capacity M after first-order correction.

μ_a, δ_a average value of maximum crack width obtained after first order correction, coefficient of variation

a_0 - maximum crack width value of real members.

$\rho_{\ln M, \ln a}$ -- the correlation coefficient between ln M and $\ln a$ after correction is determined by the equation

$$\rho_{\ln M, \ln a} = \frac{\ln\left(1+\rho_{Ma}\delta_M\delta_a\right)}{\sqrt{\ln\left(1+\delta_M^2\right)\ln\left(1+\delta_a^2\right)}} \tag{39}$$

Where

δ_M, δ_a -the coefficient of variation of the flexural member's bearing capacity M, the maximum crack width a after first correction

ρ_{Ma} the correlation coefficient between the flexural member's bearing capacity M and the maximum crack width a after correction, which is determined by formula (24) (28) (36) respectively, according to the specific case.

Bearing capacity correction criteria

Once the distribution of the corrected bearing capacity M is obtained, the revision criterion of the bearing capacity M can be obtained, and the correction criterion value M_K is given by the following equation.

$$M_k = \exp\left(\mu_{\ln M|a=a_0} - k_{\ln M|a=a_0}\right) \tag{40}$$

Where

$$\mu_{\ln M|a=a_0} = \ln_{M|a=a_0} - \frac{1}{2}\ln\left(1+\delta_{M|a=a_0}^2\right)$$

$$\sigma_{\ln M|a=a_0} = \sqrt{\ln\left(1+\delta_{M|a=a_0}^2\right)}$$

$\mu_{M \mid a=a_0}$ -mean value after correcting the bearing capacity M

$\delta_{M \mid a=a_0}$ -coefficient of variation after correcting the bearing capacity M

k –constant, 1.645 is taken when the assurance ratio of the k-constant, bearing capacity criterion is 95%.

Case example analysis

Examples: When the cross-sectional dimensions for a single beam of reinforced concrete in a public building is b × h=250 mm×500 mm, the environmental classification belongs to category 1, the cover layer thickness is c=25 mm, C25 and j 240 reinforcement bar is reinforced for the concrete strength, the cross-sectional area is A=1256 mm 2 (420), the calculated span of the beam is $l_0 = 6m$,, and the ultimate deflection value at the mid-span is $l_0 / 200$.

The reference values of the uniform distribution of dead load is $S_{Qk} = 50kNm$, the reference value of the uniform distribution of live load is φ_q =0.5.

When the crack widths are a_0 =0.16 mm, 0.20 mm and 0.24 mm, respectively, the method considering only the correlation of maximum crack width and bearing capacity is compared with the new method as follows :

Statistical variables of maximum crack width and bearing capacity in the new method considering geometric dimensions and results of concrete compressive strength measurements

$\mu_M = 133214676.6$ Nmm $\quad \sigma_M = 12000426.42$ Nmm

$\mu_a = 0.2637$ mm $\quad \sigma_a = 0.1114$ mm

$\delta_M = 0.090 \quad \delta_a = 0.4225$

$\rho_{Ma} = \text{-}0.263 \quad \rho_{\ln M \ln a} = \text{-}0.276$

Table 3. calculation results of the new method

a_0 (mm)	$\mu_{M\|a=a_0}$ (Nmm)	$\sigma_{M\|a=a_0}$ (Nmm)	M_k (Nmm)	$\delta_{M\|a=a_0}$
0.16	126273037	10931181.1	109133855.2	
0.20	124559180	10782816.2	124559180.1	0.0865
0.24	123176139	10663089.4	123176139	

Statistical variables considering only maximum crack width and bearing capacity correlation analysis

$\mu_M = 131128199.6$ Nmm $\quad \sigma_M = 12056612.14$ Nmm

$\mu_a = 0.263$ mm $\quad \sigma_a = 0.114$ mm

$\delta_M = 0.091 \quad \delta_a = 0.4338$

$\rho_{Ma} = \text{-}0.277 \quad \rho_{\ln M \ln a} = \text{-}0.292$

Table 4. Calculation results of the original method

a_0 (mm)	$\mu_{M\|a=a_0}$ (Nmm)	$\sigma_{M\|a=a_0}$ (Nmm)	M_k (Nmm)	$\delta_{M\|a=a_0}$
0.16	124251413	10924465.1	107136419.7	
0.20	122475471	10768320.3	122475471.3	0.0879
0.24	121043280	10642398.8	121043280.4	

The results of the comparative analysis of the two methods are shown in Table 5.

Table 5. Comparative analysis of the two methods

Method	a_0 (mm)	M_k (Nmm)	$\delta_{M\|a=a_0}$
	0.16	107136419.7	
Original method	0.20	122475471.3	0.0879
	0.24	121043280.4	
	0.16	109133855.2	
New method	0.20	124559180.1	0.0865
	0.24	123176139	

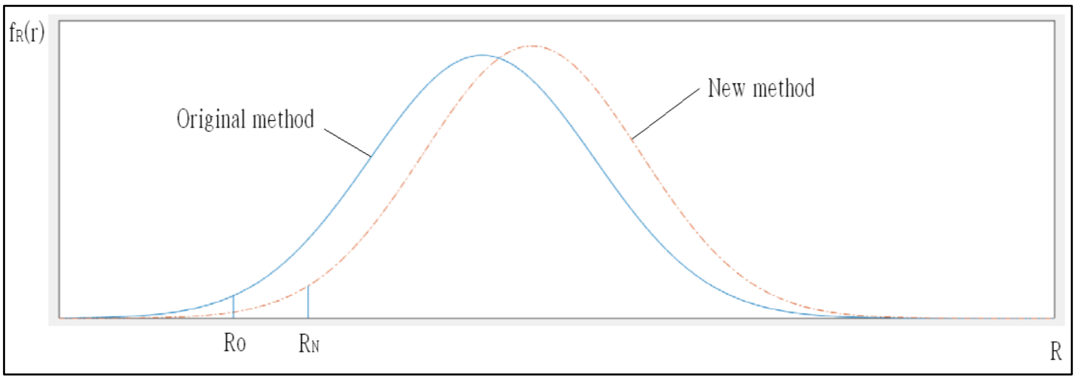

Figure 1. Variation of the bearing capacity probability density function curve before and after revision.

4. Conclusions

In summary, accurately assessing the bearing capacity of existing building structures is essential to maintaining the safety and extending the economic lifespan of these structures. This study investigated and compared two assessment methods for reinforced concrete beams: the traditional approach, which primarily considers the correlation between maximum crack width and bearing capacity, and a newly developed probabilistic approach that incorporates the probability distribution characteristics of key variables influencing bearing capacity.

The findings demonstrate that the new probabilistic method provides a more comprehensive and scientifically grounded approach, enabling refined calculations that account for the inherent variability in technical assessment data. This approach allows for a more rational determination of bearing capacity, improving the reliability of safety evaluations and supporting optimized maintenance and reinforcement strategies for aging structures. The proposed method thus contributes to a framework for enhanced structural integrity assessments, promoting safer and more sustainable building practices.

5. Acknowledgements

The authors would like to express their sincere gratitude to Pyongyang University of Architecture for providing essential resources and support for this study. Special thanks are extended to the faculty members and research colleagues for their invaluable insights and assistance in data analysis and model verification. Additionally, we acknowledge the contributions of the technical staff involved in the field assessments, whose efforts were fundamental to the successful execution of this research. We are also grateful for the constructive feedback provided by anonymous reviewers, which helped improve the quality and clarity of this paper.

6. Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper. This research was conducted independently, and no financial, commercial, or personal relationships influenced the study outcomes. Any affiliations or resources provided by the authors' institutions served purely academic purposes and did not affect the findings or interpretations presented herein.

References

[1] J. Smith dan A. Brown, "Reliability Assessment of Existing Structures: Challenges and Approaches," Journal of Structural Engineering, vol. 146, no. 4, pp. 1-12, Apr. 2020.
[2] M. T. Yang dan L. H. Chen, "Comparative Study on Reliability Analysis Methods for New and Existing Structures," Engineering Structures, vol. 215, pp. 1-10, Mar. 2020.
[3] R. K. Gupta dan S. P. Sharma, "Building Management Strategies for Enhanced Reliability and Durability," Construction and Building Materials, vol. 244, pp. 1-15, Jul. 2020.
[4] International Standard. Bases for Design of Structures-Assessment of Existing structures(ISO13822:2003). 2003.
[5] International Standard. General principles on reliability for structures ISO2394:1998), 1998.

[6] Yao, James T P, Natke H G. Damage detection and reliability evaluation of existing structures. Structural safety,1994

[7] A. Kudzys and O. Lukoševičienė, "On the safety prediction of deteriorating structures," Mech., vol. 78, pp. 5-11, 2009.

[8] I. Skrzypczak, M. Słowik, and L. Buda-Ożóg, "The Application of Reliability Analysis in Engineering Practice – Reinforced Concrete Foundation," Procedia Engineering, vol. 193, pp. 144-151, 2017.

[9] M. Sýkora, M. Holicky, and J. Marková, "Target reliability levels for assessment of existing structures," 2014.

[10] J. Marková, M. Holicky, K. Jung, and V. Bureš, "Reliability assessment of cast-iron heritage structures," 2014.

[11] M. Sýkora, D. Diamantidis, M. Holicky, and K. Jung, "Target reliability for existing structures considering economic and societal aspects," Structure and Infrastructure Engineering, vol. 13, pp. 181 - 194, 2017

[12] E. Verstrynge, C. Van Steen, and M. Wevers, "Acoustic emission testing for multi-scale assessment of reinforcement corrosion in concrete structures," e-Journal of Nondestructive Testing, 2023.

[13] D. Val and M. Stewart, "Reliability Assessment of Ageing Reinforced Concrete Structures—Current Situation and Future Challenges," Structural Engineering International, vol. 19, pp. 211-219, 2009.

[14] C. Li and R. Melchers, "Time-Dependent Risk Assessment of Structural Deterioration Caused by Reinforcement Corrosion," ACI Structural Journal, vol. 102, pp. 754-762, 2005.

[15] V. Nadolski, M. Holicky, M. Sýkora, and V. Tur, "Comparison of approaches to reliability verification of existing steel structures," Budownictwo i Architektura, 2022.

[16] Su Jian-kang, "Reliability-Based Capacity Assessment of Existing Bridges," Engineering Mechanics, 2010.