MATHEMATICAL MODELING AND SIMULATION OF FLUID DYNAMIC IN THE CONTINUOUS STIRRED TANK

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Abstract. This study examines the fluid dynamic models of stirring water continuously in the tank. The model is derived by using a Navier-Stokes equation. It is considered the energy balance and linearization. Simulation is built by using the Navier-Stokes equation. In the results and discussion has been developed two simulation cases, i.e. in the speed of 10rpm and 20 rpm. From the results and discussion it can be concluded that the velocity magnitude distributions in the case of speed variation are different. In the speed of 10rpm the velocity distribution more irregular which compared with the speed of 20rpm.

1. INTRODUCTION

Crude palm oil is a foreign exchange earner for Indonesia mainstay of the plantation industry. It can be seen from the CPO market share with approximately 80% of palm oil on the market is the world produced by Indonesia and Malaysia. Various attempts have been made to improve the performance of the palm oil industry. One of them is through the garden-livestock integration system or known as system integration palm-cow (SISS). This system has been proven to increase productivity and farmers’ welfare in
which profits not only from the plantation sector but also from the livestock sector. However, the problem of waste is still a major obstacle in the CPO industry sector. On the other hand, the wastewater treatment system of palm oil mill is one of the major sources of emissions of greenhouse effect gases (GHG) [1].

Waste generated from palm oil industries including heavy waste categories with high quantity and content of contaminants that can reach up to 20000-60000 mg/l and 40000-120000 mg/l respectively for BOD (biochemical oxygen demand) and COD (chemical oxygen demand). Waste is mainly generated from the boiling stage, precipitation, decantation, and centrifugation were carried out during the clarification process CPO. Wastewater produced from this stage no less than 2.5 m$^3$/ton CPO products. If the projected production of 10 million tons launched reached, it will produce approximately 25 million m$^3$ of wastewater. Waste in the form of heavy condensate phase and thus the characteristics are very difficult to overcome only with the concept of end-off pipe treatment alone. Major issue then is where the technological constraints of existing wastewater treatment is currently difficult to produce output that leads to a waste-free CPO industry [19].

The competitiveness of an industry is not only determined by the number, quantity, and price of the products, but is also determined by the production process used primarily for the production of export-oriented. Moving on from the problems encountered in the field, integrated solutions from program zero waste effluent and farm-livestock integration in the palm oil industry is a very attractive alternative to resolve the issue of industrial waste.

According to the information from the Ministry of Agriculture of Indonesia, the issue of renewable energy has become a huge talking point in Indonesia at this time, began to decrease in reserves related to energy derived from petroleum. Other energy sources must be sought with the use of natural resources in Indonesia alone. One of the beneficiaries of the resources that can be used is the utilization of palm oil mill effluent (POME). With oil palm plantations which reached seven million hectares in 2011, with productivity 3595Kg/Ha [5]. Obviously this needs to be developed POME utilization.

Research on POME conversion into biogas has been done by Irvan et al. [18]. In this study, the conversion is done with the help of anaerobic microbes in a continuous stirred reactor with a capacity of 2 liters at a temperature of 55°C, with a closed system, and the feedback input is intermittent. In
another section, Irvan et al. [18] have also successfully done the conversion on a pilot scale using fermenter capacity of 3000 liters at a temperature of $55^\circ C$, and a closed system. With a hydraulic retention time of 25, the gas produced approximately $3 \text{ m}^3$, which is equivalent to 25 liters for every 1 liter POME fed.

Garrido et al. [12, 14] have examined the related simulation phenomenological theory of sedimentation-consolidation processes for continuous batch settling and thickening of the suspension. Results obtained in the form of two alternatives. Input a solid rate and concentration flux flow downward. If the thickener widely known, the capacity and the concentration profiles in the equipment can be predicted.

2. MATHEMATICAL MODELING

In the problem of continuous stirred tank reactor, Mathematical modeling of a continuous stirred tank reactor numerically can be derived from some of assumptions: a complete mixture in the reactor and its cover, and a constant reactor volume and its cover.

The mathematical model of the process as formulated by using the mass that out from the tank and an energy balance, and by considering constitutive equation.

2.2. Mass balance in the reactor

Mass balance equation of reactor is denoted as the following equation:

$$V \frac{dC_P}{dt} = q(C_{P0} - C_P) - V r_P$$  \hspace{1cm} (1)

with $C_P$ is the concentration of the component product $P$ in the reactor, and $r_P$ is the speed of reaction per unit volume. Reaction product in first order is written in the following equation,

$$r_P = k_0 \exp \left( \frac{-E}{RT} \right) C_P$$  \hspace{1cm} (2)

where $k_0$ is a constant of reaction speed, $E$ is activation energy, $R$ is ideal gas constant, and $T$ is the reactor temperature in the absolut scale ($R$, Rankine or $K$, Kelvin).
2.3. Reactor Balance Energy

Reactor energy balance is denoted by using the following equation:

\[ V \rho C_p \frac{dT}{dt} = q \rho C_p (T_0 - T) - (\Delta H) V r_p + \rho_c C_{pc} \left[ 1 - \exp \left( \frac{-h_p}{q_c \rho_c C_{pc}} \right) \right] (T_{c0} - T) \]  

(3)

where \((-\Delta H)\) is reaction heat, \(h_P\) is coefficient of heat transfer, \(T_0\) is feeding temperature, and \(T_{c0}\) is the coolant temperature in the inlet. From the equation 1 - 3, the mass balance and energy balance equation of continuous stirred tank reactor is written as follows:

\[ \frac{dC_P}{dt} = \frac{q}{V} (C_{P0} - C_P) - k_0 C_P \exp \left( \frac{-E}{RT} \right) \]  

(4)

\[ \frac{dT}{dt} = \frac{q}{V} (T_0 - T) - \left( \frac{\Delta H}{\rho C_p} \right) k_0 C_A \exp \left( \frac{-E}{RT} \right) 
+ \frac{\rho_c C_{pc}}{\rho C_p V q_c} \left[ 1 - \exp \left( \frac{-h_p}{q_c \rho_c C_{pc}} \right) \right] (T_{c0} - T) \]  

(5)

The model equation of continuous stirred tank reactor as written in the equation 4 and 5 consist a nonlinear function in \(T\) and \(C_P\). They integrated and it is not possible to solve one equation independently to another. To design a controller for such nonlinear process, it can be used a system of locally linear model.

2.4. The form of State Variables of Equation

The equation 4 and 5 are denoted in the state variable standard form as follows.

\[ \frac{dC_P}{dt} = f_1(C_P, T) = \frac{q}{V} (C_{P0} - C_P) - k_0 C_P \exp \left( \frac{-E}{RT} \right) \]  

(6)

\[ \frac{dT}{dt} = f_2(C_P, T) = \frac{q}{V} (T_0 - T) - \left( \frac{\Delta H}{\rho C_p} \right) k_0 C_A \exp \left( \frac{-E}{RT} \right) 
+ \frac{\rho_c C_{pc}}{\rho C_p V q_c} \left[ 1 - \exp \left( \frac{-h_p}{q_c \rho_c C_{pc}} \right) \right] (T_{c0} - T) \]  

(7)
3. RESULTS AND DISCUSSION

3.1. Simulation using COMSOL Multiphysic

Simulation is performed using COMSOL as follows. Two cylinders are generated coincident. The inner domain is represented by a rotating (apar-]
\begin{align*}
\frac{\partial u}{\partial t} + \rho (u \cdot \nabla) u &= \nabla \cdot \left[ \rho i + \mu \left( \nabla u + (\nabla u)^T \right) - \frac{2}{3} \mu (\nabla \cdot u) i \right] + F \\
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) &= 0
\end{align*}

The wall is assumed in the model as in the Equation 9, and the rotating wall equation as in Equations 10 and 11. The rotating domain is using the
Equation 12. In the model it is also assumed the symmetry for upper of domain as in the Equations 13 - 15.

\[ u = 0 \quad (9) \]
\[ u = v_{\text{wall}} \quad (10) \]
\[ \left. \frac{\partial x}{\partial t} \right|_x = v_{\text{wall}} \quad (11) \]
\[ dx = dx(r_{bp}, \omega, t) \quad (12) \]
\[ u \cdot n = 0 \quad (13) \]
\[ K - (K \cdot n)n = 0 \quad (14) \]
\[ K = \left[ \mu (\nabla u + (\nabla u)^T) \right] n \quad (15) \]

3.2. Mesh

By using Finite Element Method, the mesh is generated using tetrahedron as depicted in Figure 2. A careful selection of the mesh type and number is considered before the simulation. These have a significant influence for calculation accuracy and efficiency. In 90% of the overall volume of the CSTR hexahedron mesh type is used in the meshing scheme for its higher calculation accuracy in 3-dimension multi-phase simulation.

3.2. Velocity Magnitudes

Case: Speed 10 rpm

The velocity magnitudes for the speed of 10 rpm are vary in the slice of the blade level. It shows a clockwise directed circulation pattern governing most of the cross-section. Only flow areas before and behind baffles show flow distortions and a low average velocity. The central region below the area swept by the moving impeller shows an intense tangential circulation flow. Outside the central region the tangential velocity component is decreasing and passing into a stronger radial velocity component.

Case: Speed 20 rpm

The velocity magnitudes for the speed of 20 rpm are more regularly in the slice of the blade level.
3.3. Velocity Fields

4. CONCLUSION

It has been built a model of fluid dynamic of stirred water in a continuous stirred tank. Simulation is built by using the Navier-Stokes equation. In the results and discussion has been developed two simulation cases, i.e. in the speed of 10rpm and 20 rpm. From the results and discussion it can be concluded that the velocity magnitude distributions in the case of speed variation are different. In the speed of 10rpm the velocity distribution more irregular which compared with the speed of 20rpm.
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Figure 5: Velocity Field speed 10 rpm


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