B-SPLINE APPROXIMATION FOR IMAGE PROCESSING

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Abstract. This paper is to show the research results according to refine an image size enlargement. B-spline approximation based on a two dimensional quadratic interpolation is used to approximate the image color value after enlargement processes. A procedure is performed on every $3 \times 3$ pixels in the image surface in the form of matrices. Smoothing iterations is also performed twice to obtain better results. A quadratic function is used as transformation. Matlab programming is applied to the JPEG image of size of $30 \times 30$ and $75 \times 98$ pixels that are doubled magnified. The results show that this method can smooth the images when the size is enlarged.

1. INTRODUCTION

Image processing problem (which is usually known as a digital image) is one part of the study in Informatics and is still studied by many researchers. The goal of research in this area is generally to improve the quality of the image, or also to provide additional extra-picture effect to be more attractive. The methods that have been developed in the image processing are also quite a lot, such as edge detection. There was also a researcher who perform image processing by combining the knowledge of image search [1, 2]. While Chen et al. [3] present data structures-new bilateral grid, that enables...
image processing edge-aware are fast, and can be applied to image editing, transfer of photographic look, and contrast enhancement of medical images.

Research in the field of image processing is carried out by some researchers in the last decade. There are techniques have been used which depend on the objective of the image processing. Usually, some transformations are performed to get a new desired image. Scarano and Riethmuller [4] have proposed an image processing technique by using iterative interrogation of particle image velocimetry (PIV) recordings and applied progressive grid refinement in order to have a maximum spatial resolution.

B-spline (bell shaped spline) is a special class of spline, which is a function of rate cut to pieces many of which are connected by a continuous curve segments smaller attention to continuity that have a direct result on the basis functions. Debral et al. [5] have used cubic B-spline in resolving a problem using the finite element method. In the research with respect to hierarchical approach, Voung et al. [6] has performed a local refinement for isogeometrical analysis, that is done based on a hierarchical B-Spline extension. In the research it was observed the theoretical properties of spline space according to satisfy the fundamental properties. On the other hand, the finite element analysis is used to integrate the hierarchical spline space. The objective of this research is to analyse the process of quadratic B-Spline procedures in the refinement of some types of images.

2. B-SPLINE QUADRATIC FORMULATION

A spline can be used to solve some problems in which an estimation is computed using some higher degree polynomials that come from many given data points [7]). A spline is described as a bending thin plate attached to each data points that are analogous to the pin. Figure 1 illustrates this mechanism [6].

![Figure 1: Spline model mechanism](image)

A spline function of \( p \)-order to each subject \( i \) in order to estimate its underlying smooth, \( f_i \). Let \( t \in [a, b] \), where \( a, b \in \mathbb{R} \) and \( x_0 = a < x_1 < \)
\[\cdots < x_L < b = x_{L+1}\] be a subdivision on the interval \([a, b]\) by \(L\) distinct points, termed internal knots. The knot sequence is augmented by adding replicates at the beginning and end, \(\tau = [\tau_1, \cdots, \tau_{L+2p}]\) for \(p \in \mathbb{N}\), such that

\[
\tau_1 = \tau_2 = \cdots = \tau_p = x_0
\]
\[
\tau_{j+p} = x_j, \quad j = 1, \cdots, L
\]
\[
x_{L+1} = \tau_{L+p+1} = \tau_{L+p+2} = \cdots = \tau_{L+2p}
\]

A useful basis \(B_1^p(t), \cdots, B_{L+2p}^p(t)\) for this linear space is given by Schoenberg’s B-splines [8], and its B-spline curve can be define as follows [9].

**Definition 1** Let \(t = (t_0, t_1, \cdots, t_n)\) be a knot vector. B-spline function of \(k\) degree is defined as

\[
N_i^0(t) = \begin{cases} 
1 & \text{if } t \in [t_i, t_i+1) \\
0 & \text{otherwise} 
\end{cases} \quad (1)
\]

\[
N_i^k(t) = \frac{t - t_i}{t_{i+k} - t_i} N_i^{k-1}(t) + \frac{t_{i+k+1} - t}{t_{i+k+1} - t_{i+1}} N_{i+1}^{k-1}(t) \quad (2)
\]

where \(0 \leq i \leq n - k - 1, 1 \leq k \leq n - 1, 0 \leq 0 := 0\).

**Definition 2** Let \(P_0, P_1, \cdots, P_m\) \((P_i \in \mathbb{R}^d)\) be \(m + 1\) control points, \(t = (t_0, t_1, \cdots, t_{m+n+1})\) knot vector. B-spline curve of \(n\) degree for control points \(P_i\) and knot vector \(t\) is defined as

\[
C(t) = \sum_{i=0}^{m} P_i N_i^n(t) \quad (3)
\]

where \(N_i^k\) are base B-spline function from definition 1.

Joy [10] derived the matrix equation for biquadratic spline surface as a control nodes, as depicted in figure 2. This matrix can be used to compute the value of a node from the given some values of some nodes. A biquadratic uniform B-spline surface \(P(u, v)\) is defined by the \(3 \times 3\) array of control nodes, as follows.

\[
P = \begin{bmatrix} P_{0,0} & P_{0,1} & P_{0,2} \\
P_{1,0} & P_{1,1} & P_{1,2} \\
P_{2,0} & P_{2,1} & P_{2,2} \end{bmatrix} \quad (4)
\]
By using the control nodes in the equation 4, the components of $P(u, v)$ are determined by the equation

$$P(u, v) = \begin{bmatrix} 1 & u & u^2 \end{bmatrix} MPM^T \begin{bmatrix} 1 \\ v \\ v^2 \end{bmatrix}$$  \hspace{1cm} (5)$$

$$P_{0,0}, P_{0,1}, P_{0,2} \quad P_{1,0}, P_{1,1}, P_{1,2} \quad P_{2,0}, P_{2,1}, P_{2,2}$$

Figure 2: Quadratic spline

A new surface $P'(u, v)$ is determined by the following equation

$$P' = \frac{1}{4} \begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} P_{0,0} & P_{0,1} & P_{0,2} \\ P_{1,0} & P_{1,1} & P_{1,2} \\ P_{2,0} & P_{2,1} & P_{2,2} \end{bmatrix} \frac{1}{4} \begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 3 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} P'_{0,0} & P'_{0,1} & P'_{0,2} \\ P'_{1,0} & P'_{1,1} & P'_{1,2} \\ P'_{2,0} & P'_{2,1} & P'_{2,2} \end{bmatrix}$$  \hspace{1cm} (6)$$

where $P'_{i,j}$ is written as

$$P'_{0,0} = \frac{1}{16}(3(3P_{0,0} + P_{1,0}) + (3P_{0,1} + P_{1,1}))$$
$$P'_{0,1} = \frac{1}{16}((3P_{0,0} + P_{1,0}) + 3(3P_{0,1} + P_{1,1}))$$
$$P'_{0,2} = \frac{1}{16}(3(3P_{0,1} + P_{1,1}) + (3P_{0,2} + P_{1,2}))$$
$$P'_{1,0} = \frac{1}{16}(3(P_{0,0} + 3P_{1,0}) + (P_{0,1} + 3P_{1,1}))$$
3. THE IMPLEMENTATION FOR IMAGE REFINEMENT

This study is designed to refine some bitmap images that have become coarse after enlargement process. The research process is performed in the following steps:

(1) An image with the dimension of \( m \times n \) is given and the the values of RGB are read node by node.

(2) The image is enlarged into \( 2m \times 2n \).

(3) For each \( 3 \times 3 \) pixel node is transformed by using equation 6, that is applied to each component in RGB.

(4) The whole process are computed twice to produce a good refinement.

Programming using Matlab have been developed refer to the above process flow. First of all, the input image is read into some array type variables. The enlargement is performed, and then the results are saved into another array variables. The interpolation using B-spline are performed for each values of RGB to get new values that develop new more refine image.

4. RESULTS AND DISCUSSION

The implementation of the process have been applied to two types of images (from camera captured and from painting) with the dimension of \( 30 \times 30 \) and \( 75 \times 98 \). Figure 3 shows an image of \( 30 \times 30 \) that come from camera captured that have been enlarged into \( 60 \times 60 \). The original image that
is developed by some forming pixels $1 \times 1$ turn into the the image that is developed by some forming pixels $4 \times 4$. On the other hand, figure 4 shows that the image is more smoothly.

The second image is depicted in figure 5. The image is of the dimension $75 \times 98$ that comes from painting and has been enlarged into $150 \times 196$. Figure 6 shows that the image is also more smoothly after enlargement process.

To analyze the transformation of the images some sample points are read from 5. A matrix of $4 \times 5$ the crop of coordinate position $7 \leq i \leq 10, 6 \leq j \leq 10$ for the red component of RGB, as shown in Table 1. After enlargement process, the result is a matrix of red component that have the dimension of $8 \times 10$ that correspondence to the new image $13 \leq i \leq 20, 11 \leq j \leq 20$, as shown in Table 2. The table shows that the matrix red component
Figure 5: Image with dimension $75 \times 98$ and $150 \times 196$

Figure 6: Image with dimension $150 \times 196$ from the refinement process

is same of all for each sub matrix $2 \times 2$ that are the copies of the matrix from the previous matrix.

Table 1: The sample of the red component from the painting

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Table 2: Red component of the matrix before enlargement

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Table 3 shows the component matrix from the first refinement that come from interpolation process by B-spline. From the interpolation it is obtained the change of the color value expressed in the matrix component. It is seen that the pixel color values (13, 12) which previously was 189 which equals the value of the pixel (13, 11) changed to 186. The value is the result of B-spline which is computed from the value of the surrounding pixels components, where there is value 175 at pixel (13, 13). The same thing happened to another pixel that is done the same way. Further refinement is obtained from previous refining as in Table 4.

Table 3: Red component values after first refinement

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Matrix of pixel component values that is stated in Table 1 - 4 can be expressed in mesh to view the fineness graphically. Figure 5 shows the change
Table 4: Red component values after second refinement

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in value due to the magnification of the image and thereafter smoothed. From this picture it appears that the image becomes smoother after the refining process using interpolation.

From the analysis to the matrices of red components, it can be concluded that the refinement can be performed using this interpolation.

5. CONCLUSION

The discussion above has shown that quadratic B-spline can be used for refining an image of bitmap type after enlargement process. B-spline originally was applied in the one dimension domain, but for the application in the image processing, it can be extended into two dimension.

References


Figure 7: Sample of original image, enlargement and processing result


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