Fuzzy Neighborhood of Cluster Centers of Electric Current at Flat EEG during Epileptic Seizures

Muhammad Abdy and Tahir Ahmad

Abstract. Flat EEG is a result of the transformation of the EEG signal to the form of two-dimensional. In this paper, we construct a fuzzy neighborhood of each cluster center of electric current on the flat EEG. Flat EEG was formed in the pixels, and each pixel has a membership value to a cluster. The pixels form a fuzzy set and it will be called as fuzzy neighborhood from a cluster.

1. INTRODUCTION

Electroencephalography (EEG) is a recording of electrical activity of the brain from the scalp and is non-invasive in nature. It plays an important diagnostic role in epilepsy and provides supporting evidence of a seizure disorder as well as assisting with classification of seizures and epilepsy syndrome. Seizures are the result of sudden excessive electrical discharges in a group of brain cells (neurons). Epilepsy describes the condition of a patient having recurring "spontaneous" seizures or brainstorm due to the sudden development of synchronous firing in the cerebral cortex caused by lasting cerebral abnormality. Surgical treatment may be an option in patient having epileptic seizures refractory to medication [1]. EEG has been used

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extensively to record the abnormal brain activity associated with epileptic seizures. The type of activity and the area of the brain that is recorded from EEG will assist the physician in prescribing the correct medication for certain type of epilepsy [2]. EEG analysis still relies mostly on its visual inspection. Due to the fact that visual inspection is very subjective and hardly allows any statistical analysis or standardization, several methods were proposed in order to quantify the information of the EEG. Among these, the Fourier transform emerged as very powerful tool capable of characterizing the frequency component of the EEG signal. However, Fourier transform has some disadvantaged that limit its applicability and therefore, other methods to extract hidden information from EEG signal are necessary [5].

2. FLAT EEG

Fauziah [3] has developed a new method to map high dimensional signal, namely EEG signal into low dimensional space (MC). The process is a transformation of EEG signal originating from the patient’s head into two-dimensional planes (MC). The process consists of three main parts. The first part is flattening the EEG where the transformation of three dimensional space into two dimensional space that involved location of sensor in patient’s head with EEG signal. The second part is the EEG signal is then processed using Fuzzy c-Mean (FCM). The last part is to find the optimal number of cluster by using cluster validity analysis. The result from the transformation of the EEG signal into low-dimensional space is called as flat-EEG (fEEG). fEEG has been used purely for visualization, but the main scientific value will lie in the ability of flattening method to preserve information about the surface. Fauziah’s EEG coordinate system (Figure 1 a) is defined as

\[
C_{EEG} = \{ ((x, y, z), e_p) : x, y, z, e_p \in \mathbb{R} \text{ and } x^2 + y^2 + z^2 = r^2 \} \quad (1)
\]

where \( r \) is the radius of a patient head. The human’s head is modeled as a sphere.

Furthermore, the mapping of \( C_{EEG} \) to a plane (MC) is defined as follows. \( S_t : C_{EEG} \rightarrow MC \) (see Figure 1b) such that

\[
S_t ((x, y, z), e_p) = \left( \frac{rx + i ry}{r + z}, \frac{ry}{r + z}, e_p \right)_{e_p(x, y, z)} \quad (2)
\]

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\[
MC = \{ ((x, y)_0, e_p) : x, y, e_p \in \mathbb{R} \} \quad (3)
\]
Both $C_{EEG}$ and $MC$ are designed and proven as 2-manifolds. Meanwhile $S_t$ is designed to be a one to one function as well as being conformal. Details of proofs are contained in [3]. Therefore, the mapping can preserve information from the surfaces of high dimensional space into a low dimensional space ($fEEG$). The EEG signal during seizure as Figure 2 (a) can be analyzed second by second at $fEEG$ as Figure 2 (b).

The flattening method has been coded as a software and received copyright ©2006-2010 Universiti Teknologi Malaysia - All Rights Reserved recently.

3. FUZZY NEIGHBORHOOD

Our aim in this section is to construct fuzzy neighborhood of current sources at $fEEG$. Let $C$ be all the cluster centers at time $t$, i.e.:
$C_t = \{c_1, c_2, \ldots, c_m\}$; $m = \text{the number of cluster center at time } t$. Each $c_j$ carries the position a $f$EEG and electrical potential, in short: $(c_j)_t = ((x, y), V_j)_t$; $V_j = \text{the electrical potential of the cluster center } j^{th}$. so, $C_t = \{((x, y), V_j)_t \mid x, y, V_j \in \mathbb{P}\}$; where $j = 1, 2, \ldots, m$. Let $\mathbb{P}$ be the entire pixels of $f$EEG, i.e.: $\mathbb{P} = \{p_1, p_2, \ldots, p_n\}$ such that $p_i = (x, y)$, then $C_t \subset \mathbb{P}$.

We consider a fuzzy neighborhood of electrical potential for cluster center $c_j$ at each pixel $p_i$ at time $t$. The fuzzy neighborhood is represented by a membership function $\mu_{c_j}$. This membership function must satisfy several properties.

Considering that $p_k$ and $p_l$ are two pixels of the $f$EEG, then:

(i) $\forall p_k, p_l \in \mathbb{P}$ and $|p_k - c_j| = |p_l - c_j|$ iff $\mu_{c_j}(p_k) = \mu_{c_j}(p_l)$

(ii) $\forall c_j \in C, \mu_{c_j}(c_j) = 1$

We propose membership function $\mu_{c_j}$ at time $t$, as:

$$\mu_{c_j}(p_i)_t = \frac{(V_j)_t}{(V_j)_t + d(p_i, c_j)_t} \quad (4)$$

where $(V_j)_t$ and $d(p_i, c_j)_t$ are the electrical potential at $c_j$ and Euclid distance between pixel $p_i$ and cluster center $c_j$ at time $t$, respectively. We can show easily that (4) satisfies both properties.

**Theorem 1**

Let $(V_j)_t$ and $(V_w)_t$ be electrical potential at cluster center $(c_j)_t$ and $(c_w)_t$, respectively, and $p_k, p_l \in \mathbb{P}$. If $(V_j)_t \geq (V_w)_t$ and $d(p_k, c_j)_t = d(p_l, c_w)_t$ then $\mu_{c_j}(p_k)_t \geq \mu_{c_w}(p_l)_t$.

**Theorem 2**

Let $(V_j)_t$ and $(V_w)_t$ be electrical potential at cluster center $(c_j)_t$ and $(c_w)_t$, respectively, and $p_k, p_l \in \mathbb{P}$. If $(V_j)_t = (V_w)_t$ and $d(p_k, c_j)_t \geq d(p_l, c_w)_t$ then $\mu_{c_j}(p_k)_t \leq \mu_{c_w}(p_l)_t$.

4. IMPLEMENTATION AND DISCUSSION

We will apply at flat-EEG data from an epileptic patient. We choose time 4th as example (see Figure 3). It has three cluster centers as at Table 1. At every cluster center, the membership value is equal to one. When the pixel further away from the cluster center, the smaller the value of membership.
Figure 3 Flat EEG at time 4\textsuperscript{th}

Table 1: Position of Cluster Centers and Electrical Potential at time 4\textsuperscript{th} on \textit{f}EEG during epileptic seizures.

<table>
<thead>
<tr>
<th>Time (second)</th>
<th>Position of cluster centers</th>
<th>Electrical Potential (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4th</td>
<td>0.1634</td>
<td>3.3869</td>
</tr>
<tr>
<td>1.7033</td>
<td>-0.19834</td>
<td>329.535856457902000</td>
</tr>
<tr>
<td>-0.9222</td>
<td>-1.8426</td>
<td>25.1987293595413000</td>
</tr>
</tbody>
</table>
Table 2: Fuzzy Neighborhood for every cluster centers at time 4th on fEEG during epileptic seizures.

(a) Cluster center 1 ($C_1$)

<table>
<thead>
<tr>
<th>t</th>
<th>Cluster 1</th>
<th>Cluster 2</th>
<th>Cluster 3</th>
<th>Cluster 4</th>
<th>Cluster 5</th>
<th>Cluster 6</th>
<th>Cluster 7</th>
<th>Cluster 8</th>
<th>Cluster 9</th>
<th>Cluster 10</th>
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<tr>
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<td>0.34</td>
<td>0.23</td>
<td>0.45</td>
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<tr>
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<td>0.23</td>
<td>0.34</td>
<td>0.12</td>
<td>0.45</td>
<td>0.23</td>
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<tr>
<td>3</td>
<td>0.12</td>
<td>0.45</td>
<td>0.23</td>
<td>0.34</td>
<td>0.12</td>
<td>0.45</td>
<td>0.23</td>
<td>0.34</td>
<td>0.12</td>
<td>0.45</td>
</tr>
<tr>
<td>4</td>
<td>0.34</td>
<td>0.23</td>
<td>0.45</td>
<td>0.12</td>
<td>0.34</td>
<td>0.12</td>
<td>0.45</td>
<td>0.23</td>
<td>0.12</td>
<td>0.34</td>
</tr>
</tbody>
</table>

(b) Cluster center 2 ($C_2$)

<table>
<thead>
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<th>t</th>
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<th>Cluster 3</th>
<th>Cluster 4</th>
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<tr>
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(c) Cluster center 3 ($C_3$)

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<td>0.45</td>
<td>0.23</td>
<td>0.12</td>
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</tr>
</tbody>
</table>
5. CONCLUSION

We present a construction of fuzzy neighborhood of cluster centers at fEEG. The fuzzy neighborhood is considered to represent electric current that determines the cluster center. Our next work is to find fuzzy region on fEEG at each time (second) during epileptic seizures using our proposed model.

References


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