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Parameter Estimation of GARCH Model with Bootstrap Approximation

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Abstract. This study aims to estimate the parameters of the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model using a bootstrap approximation. In the heteroscedasticity data model, it is determined how much the residual value of the sample used is. The bootstrap approach is a non-parametric and resampling technique used to estimate the parameter. From the sample data implemented, the residual estimation using the Maximum Likelihood Estimation method is $\varepsilon_t^* = 0.065851304$. Furthermore, the residual estimation value using the bootstrap is $\varepsilon_t^* = -1.769129241$. Thus, the use of the bootstrap approach in the GARCH model results in a smaller residual value than MLE.

Keyword: Estimation Parameter, Generalized Autoregressive Conditional Heterokedasticity (GARCH), Bootstrap Approximation, Maximum Likelihood Estimation (MLE) Method.

Abstrak. *Penelitian ini bertujuan menaksir parameter model Generalized Autoregressive Conditional Heterokedasticity (GARCH) menggunakan pendekatan bootstrap. Pada model data yang bersifat heterokedastisitas ditentukan seberapa besar nilai residual dari sampel yang digunakan. Pendekatan bootstrap merupakan teknik non-parametrik dan resampling yang digunakan untuk menaksir parameter. Dari contoh data yang diimplementasikan diperoleh penaksiran residual dengan metode Maximum Likelihood Estimation adalah* $\varepsilon_t^* = -0,065851304$. *Selanjutnya nilai penaksiran residual dengan menggunakan pendekatan bootstrap adalah* $\varepsilon_t^* = -1,769129241$. *Dengan demikian penggunaan pendekatan bootstrap pada model GARCH menghasilkan nilai residu yang lebih kecil dari MLE. .*

Kata Kunci: *Penaksiran Parameter, Model Generalized Autoregressive Conditional Heterokedasticity (GARCH), Pendekatan Bootstrap, Metode Maximum Likelihood Estimation (MLE).*

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1. Introduction

Parameter estimation is a field of statistics that deals with estimating parameter values based on measured data or empirical data that has a random component [1]. Parameter estimation is classified into two, namely point estimation and interval estimation. A point estimate is an estimate of a population parameter expressed by a single number to estimate the parameter. An interval estimate is an estimator whose estimated value is expressed in an interval. An estimate is an estimate of future events using a sample of data.

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Autoregressive data analysis of time-series models must meet the assumption of constant variance (homokedasticity) which is very difficult to meet. Furthermore, arch (Autoregressive Conditional Heterokedasticity) with an average value of zero was introduced, the process of noncorrelation with conditional variance was not constant taking into account asymptotic efficiency i.e. by using a simple test on ordinary square least (OLS) residual autocorrelation but found each co-efficiency was significant at the conventional level except at the first lag [2]. Meanwhile, ARCH was developed in a general form, namely Generalized ARCH (GARCH) which has a more flexible lag distribution structure and significant coefficients at the conventional level and the estimated value of the stationary Autoregressive coefficient [3] .

Parameter estimation of GARCH model can be done using Maximum Likelihood Estimation (MLE) method and bootstrap. The MLE method requires the assumption of normality in parameter estimation which is sometimes very difficult to fulfill [4]. Meanwhile, the bootstrap does not require the assumption of normality in parameter estimation.approach bootstrap technique resampling that aims to determine the estimated standard error and confidence interval of population parameters such as mean, ratio, median, proportion, correlation coefficient or regression coefficient [5].

2. Related Work

2.1. Basic Concepts of Time Series

A time series is a set of consecutive observations in a certain time. A time period is denoted by Y_t where t refers to successive time periods. A time series is said to be stationary if the following values are met.
a. $E(Y_t) =$ constant for every t
b. $Var(Y_t) =$ constant for every t
c. $Kov(Y_t, Y_{t-1}) =$ constant for every t and $Kov(Y_t, Y_{t-1})$ is dependent on lag k. Meanwhile, the time series is said to be non-stationary if the time series fails to fulfill one or more parts of the value requirement.

2.2. Autoregressive Moving Average (ARMA)

ARMA is a combination of the AR model and the MA model. The general form of the ARMA(p,q) is as follows:

$$Y_t = \varnothing_1 Y_{t-1} + \ldots + \varnothing_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} \quad (1)$$

2.3. Autoregressive Conditional Heterokedasticity (ARCH)

ARCH model is a time series model with heteroscedasticity. The general form of the ARCH(s) model is as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{s} \alpha_i \varepsilon_{t-1}^2; \tag{2}$$

$$\sigma_1 = \frac{n\sum_{t=1}^{T} \varepsilon_t^2 \varepsilon_{t-1}^2 - \sum_{t=1}^{T} \varepsilon_t^2 \sum_{t=1}^{2} \varepsilon_{t-1}^2}{n\sum_{t=1}^{T} \sigma_{t-1}^4 - (\sum_{t=1}^{T} \varepsilon_{t-1}^2)^2} \quad dan \quad \alpha_0 = \frac{\sum_{t=1}^{T} \varepsilon_t^2}{n} - \frac{\alpha_1 \sum_{t=1}^{T} \varepsilon_{t-1}^2}{n}$$

2.4. Generalizatiom Autoregressive Conditional Heterokedasticity (GARCH)

GARCH model is a generalization of the ARCH model by developing a time-series that allows conditional variance of the ARMA process [6] The general form of the GARCH model with order (p,q) is as follows:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-1}; \tag{3}$$

$$\hat{\beta}_1 = \frac{n\sum_{t=1}^{T} y_t y_{t-1} - \sum_{t-1}^{T} y_t \sum_{t=1}^{T} y_{t-1}}{n\sum_{t=1}^{T} y_{t-1}^2 - (\sum_{t=1}^{T} y_{t-1})^2} \quad dan \quad \hat{\beta}_0 = \frac{\sum_{t=1}^{T} y_t}{n} - \frac{\hat{\beta}_1 \sum_{t=1}^{T} y_{t-1}}{n}$$

2.5. Bootstrap

The bootstrap is an approach to nonparametric statistics by resampling data. Furthermore, the results obtained in the form of parameter estimates. The following are the steps used to determine the value of variance and bias using the bootstrap.

2.5.1. Bootstrap Approximation to Estimating Variance

Step 1. Given sample of size n (with replacement) from $\{X_1, X_2, ...X_n\}$, for example $\{X_{11}, X_{22}, ...X_{33}\}$, and calculate the mean $\bar{X}_1$.
Step . Repeat step 1 independently $N-1$ more times, find $\bar{x_2}, ..., \bar{X_N}$. The bootstrap estimate of Var $bar(X)$ is

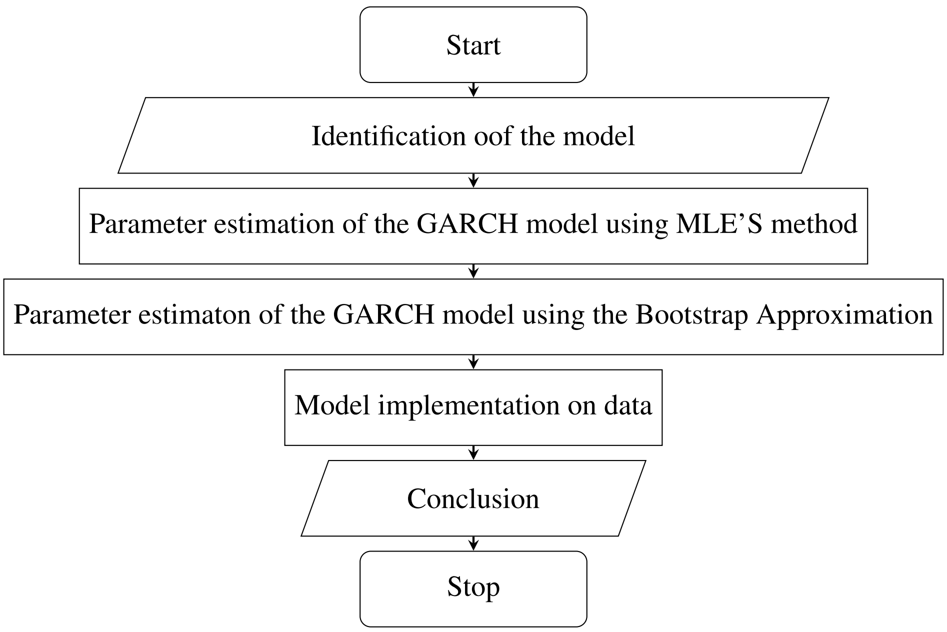
$$\hat{\sigma^2} = \frac{\sum_{i=1}^{N} (\bar{X}_i - \bar{X}_.)^2}{N-1}, \bar{X}_. = \frac{\bar{X_1} + \bar{X_2} + ... + \bar{X_N}}{N} \tag{4}$$

2.6. Maximum Likelihood Estimation Method

The maximum likelihood estimation method is one of several methods which is used to estimate the parameters. Suppose $L(\theta) = \Pi_{i=1}^{n} p_X(k_i;\theta)$ and $L(\theta) = \Pi_{i=1}^{n} f_Y(y_i;\theta)$ is the likelihood function corresponding to the random variable $k_1, k_2, ..., k_n$ dan $y_1, y_2, ..., y_n$ obtained from $p_X(k;\theta)$ and continuous functions $f_Y(y_i;\theta)$, respectively, where is an unknown parameter. In each case, suppose θ_e is a parameter value such that $L(\theta_e) \geq L(\theta)$ $\forall$ the possible values for θ. So θ_e it is called a maximum likelihood estimate for θ.

3. Methodology

At this stage, a study will be presented on how the research steps are carried out, namely the simulated data collection method. Furthermore, the method of completion and data processing (software) used will be studied, and a research framework and research plan will be presented regarding ”Evaluating the Parameters of the GARCH Model Using the Bootstrap”.



The expected results of the GARCH model parameter estimation are unbiased, consistent and efficient. Furthermore, the methods used are the MLE method and the bootstrap. Meanwhile, the data that is simulated is the Stock Return Price of PT Fortune Mate Indonesia Tbk from the official website.

4. Results and Discussion

4.1. 4.1 Parameter Estimation of the GARCH Model with Maximum Likelihood Estimation (MLE)

Method Parameter estimation using the MLE method is normally distributed with values of mean equals 0 and variance constant σ^2 [7]. From the standard distribution theory, the likelihood of ε_t is

$$\mathscr{L}_t = \Pi_{t=1}^{T}\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)exp\left(\frac{-\varepsilon_t^2}{2\sigma^2}\right) \quad atau \quad ln\mathscr{L} = -\frac{T}{2}ln(2\pi) - \frac{T}{2}ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(\varepsilon_t)^2 \quad (5)$$

Meanwhile, for sample observations it is assumed $\varepsilon_t = y_t - \beta y_{t-1}$, obtained

$$ln\mathscr{L} = -\frac{T}{2}ln(2\pi) - \frac{T}{2}ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_t - \beta y_{t-1})^2 \tag{6}$$

Futhermore, the log-likelihood derived with respect to σ^2 and β fields. Since the partial derivative is equal to maximum zero with respect to the value of $ln\mathscr{L}$ in the Ordinary Least Square (OLS) estimate for the variance and β (defined by $\hat{\beta}$ and $\hat{\sigma^2}$). Hence,

$$\hat{\sigma^2} = \sum_{t=1}^{T}\frac{\varepsilon_T^2}{T} \quad dan \quad \hat{\beta} = \frac{\sum_{t=1}^{T} y_{t-1}y_t}{\sum_{t=1}^{T} y_{t-1}^2} \tag{7}$$

4.2. Parameter Estimation GARCH Model with Bootstrap Approximation

Steps for solving the bootstrap residual by applying MLE are as follows:
Step 1 : Using the estimator $\hat{\theta}$, calculate the residual

$$\hat{\varepsilon}_t = y_t - \hat{\alpha}_0 - \sum_{i=1}^{p}\hat{\alpha}_i y_{t-i} - \sum_{j=1}^{q}\hat{\alpha}_j - \hat{\varepsilon}_{t-j}, \qquad t = 1,2,...,T \tag{8}$$

Step 2: For heteroscedasticity, calculate by

$$\hat{h}_t = \hat{\alpha}_0 + \sum_{j=1}^{s}\hat{\alpha}_j\hat{\varepsilon}_{t-j}^2 + \sum_{i=1}^{r}\hat{\beta}_1\hat{h}_{t=1}, \qquad t = 1,2,...,T \tag{9}$$

Step 3 : For $t = 1,2,...,T$ then the bias can be calculated by

$$v_t = \frac{\hat{v}_t - \hat{\mu}}{\hat{\sigma}} \tag{10}$$

for $t = 1,2,...,T$, where

$$\hat{\mu} = \tfrac{1}{T}\textstyle\sum_{t=1}^{T}\hat{v}_t \quad \text{dan} \quad \hat{\sigma^2} = \tfrac{1}{T}\textstyle\sum_{t=1}^{T}(\hat{v}_t - \hat{\mu})^2$$

Step 4 : With the empirical distribution function on $\mathscr{F}_T(x)$ pada v_t stated by

$$\mathscr{F}_T(x) := \frac{1}{T}\sum_{t=1}^{T}(v_t \leq y) \tag{11}$$

Step 5: Will result in a bootstrap Y_t^* , that is

$$Y_t^* = \hat{\alpha}_0 + \sum_{i=1}^{p}\hat{\alpha}_i Y_{t-i} + \sum_{j=1}^{q}\hat{\alpha}_i\hat{\varepsilon}_i\hat{\varepsilon}_{t-j} + \varepsilon_t^* \tag{12}$$

$$\varepsilon_t^* = \sqrt{\hat{h}_t v_t^*}, v_t^* \quad \sim^{iid} \mathscr{F}_T(y), \qquad t = 1,2,...,T$$

4.3. Parameter Estimation using Stock Return Data on the GARCH Model

To obtain good results, GARCH parameter estimation using the Bootstrap Approach will be applied to the Stock Return Price Data of PT Fortune Mate Indonesia Tbk. With daily data volatility from February 2021 to July 2021. The following data is presented in graphical form.

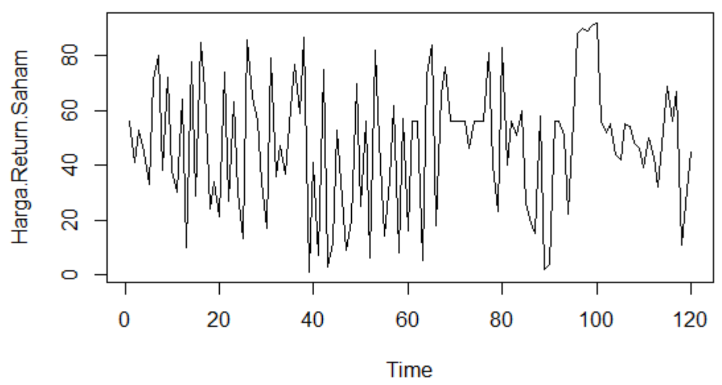


Figure 1. Plot of PT Fortune Mate Indonesia Tbk's Daily Stock Return Price (February 2021-July 2021)

From Figure 1, it can be seen that the value of the data fluctuates or it can be said that the volatility value causes the average value and variance of the data to be not constant. So that the data is not stationary (non-stationary) so that differencing that the data is stationary. Data differencing is presented in graphical form.

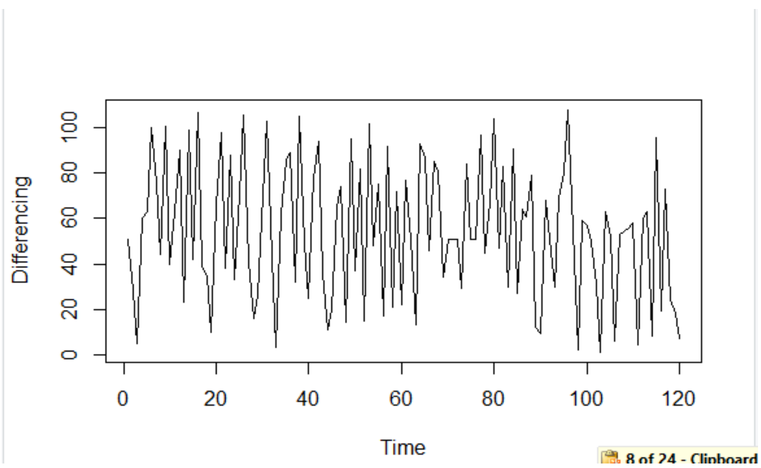


Figure 2. Plot of Stock Returns with a Difference of One (Differencing)

From Figure 2, it can be seen that after the data is centered on the average, the data spreads around the zero horizontal line, then the graph is stationary.

4.4. Parameter Estimation of GARCH(1,1) Model with Maximum Likelihood Estimation Method

Given the value $(p,q)=(1,1)$, to determine the estimated value of variance using Equation (8).

$$\hat{\sigma}^2=\frac{0,792549302}{119}=0,006607446$$

Meanwhile, to determine the residual value $\hat{\beta}$, the value of will be:

$$\hat{\beta}=\frac{-0,0602\times(-0,0661)}{0,96321874}=0,00413117$$

Futhermore, assuming that $\varepsilon_t=y_t-\hat{\beta}y_{t-1}$, will get residual

$$\varepsilon_t=-0,0661-(0,00413117\times(-0,0602))=-0,065851304$$

4.5. Parameter Estimation of GARCH(1,1) Using Bootstrap Approximation

Step 1: Calculate residual usuing Equation (9). Meanwhile, will be determined the value of $\hat{\alpha}_0,\hat{\alpha}_i$, using equation (2). Then it canbe obtained:

$$\alpha_1=\frac{119\times0,007436437-0,792893506\times0,792549302}{119\times0,021434279-(0,792549302)^2}=0,011900564,$$

next

$$\alpha_0=\frac{0,792893506}{119}-0,135786046\times\frac{0,792549302}{119}=0,010799223$$

Therefore, we get the value of

$$\hat{\varepsilon}_t=-0,0661-0,010799223-0,011900564\times(-0,0602)-0,0119005-0,01855273=-0,719359168$$

Step 2: For heteroscedasticity, it is calculated by Equation (10). Meanwhile, the value of $\hat{\beta}_0,\hat{\beta}_1$ will be determined by using Equation (3). Then it can be obtained :

$$\hat{\beta}_1=\frac{119\times-0,40500236-0,0661\times(-0,0602)}{119\times0,96321874-(-0,0602)^2}=-0,420518014,$$

next

$$\hat{\beta}_0=\frac{-0,0661}{119}-(\frac{-0,420518014\times(-0,0602)}{119})=-0,000768195$$

For $\hat{h}_{t-i}$ with $t=1$ and $i=1$ for GARCH (1,1) model, the value $\hat{h}_{t-i}=0$. Therefore, we get

$$\hat{h}_t=\alpha_0+\alpha_1\sum_{t=1}^{T}\varepsilon_{t-1}^2=0,026300994.$$

Step 3. The value of the bias estimate will be determined using Equation (11). Meanwhile, the average and variance estimation values will be calculated and obtained

$$\hat{v}_t = \frac{\hat{\varepsilon}_t}{\sqrt{\hat{h}_t}} = \frac{-0,180869631}{\sqrt{0,026300994}} = -4,435674847,$$

next

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T}\hat{v}_t = \frac{1}{119}\times(-4,435674847) = -0,037274579.$$

Therefore, we get

$$v_t = \frac{\hat{v}_t - \hat{\mu}}{\hat{\sigma}} = \frac{-0,537276044-(-0,037274579)}{0,403200692} = -10,90871211.$$

Step 4 : With the empirical distribution function on $\mathscr{F}_T(x)$ pada v_t expressed in Equation (12).

$$\mathscr{F}_T(x) := \frac{1}{119}\sum_{t=1}^{119}(-10,90871211 \leq y)$$

Step 5: The result bootstrap Y_t^* , we get by Equation (13). The value of

$$\varepsilon_t^* = \sqrt{0,026300994}\times(-10,95445115) = -1,769129241$$

Hence,

$$Y_t^* = 0,010799223+0,011900564\times(0,0602)+0,011900564\times(-0,63126574)\times(-1,766548792)$$

$$= -1,766548792$$

Furthermore, with the same calculation steps, bootstrap resampling for the values of $T = 500, T = 1000, T = 2000$, and $T = 4000$. Then we will observe the difference between the value of the GARCH model parameter estimation using the Maximum Likelihood Estimation and Bootstrap. Therefore,the value of the parameter estimation using the bootstrap and the Maximum Likelihood Estimation can be presented in tabular form.

Table 1. Parameter Estimation of ARIMA Model

T	Bootstrap			Maximum Likelihood Extimation	
	$\hat{\mu}$	$\hat{\sigma}^2$	Residu ε_t^*	$\hat{\sigma}^2$	Residu ε_t^*
500	-0,0088713	0,03869185	-3,64977908		
1000	-0,0044356	0,01934592	-5,16673899		
2000	-0,00222	0,009673	-7,31053	0,006607446	0,00413117
4000	-0,0011	0,004836	-10,3412		

5. Conclusion

Based on the results and discussion in this study, it can be concluded that the bootstrap good for this model. This is supported by the small value of the residual parameter estimation obtained from the GARCH model data used, the value of$\varepsilon_t^* = -1,769129241$, compared using the MLE method which produces a residual parameter estimation value of the $\varepsilon_t = 0,065851304$ on GARCH model.

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