Production Optimization Using Fuzzy Linear Programming Method
(Case Study: UMKM Untir-untir and Raja Manis Factory)
Esther S.M. Nababan1, 2, Agnes Purnama Sari Br. Sihaloho1, Khatimatul Husna1, Rafida Ilham1,
1Mathematics Department, Universitas Sumatera Utara, Medan, 20155, Indonesia
2Corresponding Author: esther@usu.ac.id

ARTICLE INFO
Article history:
Received: 03 July 2022
Revised: 04 August 2022
Accepted: 29 September 2022
Available online: 30 September 2022
E-ISSN: 2656-1514
P-ISSN: -

How to cite:

ABSTRACT
Production efficiency and performance optimization in a company can be achieved by using an optimization model. An optimization model can be written in a function of equation and equation known as a Linear Program. A Linear program can be combined with a fuzzy value to adjust a production problem model that is heavily dependent on an unstable supply. This modelling is known as Fuzzy Linear programming. Using Fuzzy Linear programming, the production problems of micro, small and medium-sized enterprises Untir-untir and King Sweet are analysed and solved with the help of simulation programs. Fuzzy Linear programming solutions use simplex methods using POM-QM software to complete complex calculations. Completion with the fuzzy simplex method consists of several steps namely calculating the optimal bottom and upper boundaries with the maximizing simplex technique, modifying the initial equation by adding the variable λ, and finishing it with the two-phase simplex approach. Results of completion with the simulation program showed the estimated amount of profit using the Linear Program equation of Rp. 1,378,723 and using the Fuzzy Linear program equation with a 10% tolerance of Rp. 1,447,470.

Keyword: Linear programming, Fuzzy Linear programming, Optimization

1. INTRODUCTION
Micro, small and medium enterprises play a very strategic role for the economy in Indonesia, because micro, small and medium enterprises have considerable potential to be developed. From day to day, the number of micro, small and medium enterprises is increasing as well as the competition which is progressing so rapidly, the increasing number of micro, small and medium enterprises every year has become a global issue that is hotly discussed. The rapid and difficult development and competition in the business world today encourages business people to think more critically because of the problems caused, so that every entrepreneur must develop and improve performance in order to achieve effectiveness and efficiency. So every entrepreneur also needs to look for opportunities and opportunities to compete in this increasingly advanced business world. One of them is in the food business, the food business is very much, but people are more dominant in favor of snacks or snacks. Based on data from the Central BPS, there are around 30.8% of Indonesians consuming snacks every day.

In today's competitive business environment, raw material inventory management is one of the crisis aspects for manufacturing companies to maintain a competitive advantage. However, uncertainty in the availability of raw materials is often a challenge faced by companies. Price fluctuations, delivery delays and changes in market demand can cause uncertainty in the amount of raw materials available.
To overcome this problem, companies need to develop a production optimization model that can handle uncertainty in raw material inventory, one method that can be used is Fuzzy Linear programming. This method allows companies to consider uncertainty in input parameters, such as raw material availability, and optimize production decisions to maximize profits.

This research aims to develop a Fuzzy Linear programming model for production optimization in manufacturing companies. This model will consider the uncertainty in raw material inventory and the possibility of adding raw materials during the production process, with the condition that the profit obtained remains maximized. The results of this study are expected to assist companies in making more effective and efficient production decisions, so as to increase the competitiveness and profitability of the company.

2. METHOD

2.1. Linear programming

According to Heizer and Render [1] Linear programming (LP) is a mathematical technique that is widely used to help operational managers plan and make decisions needed to allocate resources. LP is an optimization method to determine the optimum value of a linear objective function under certain constraints. These constraints are usually resource-related constraints such as raw materials, money, time, labor, etc. Linear programming problems can be found in various fields and can be used to help make decisions to choose the most appropriate alternative and the best solution [2].

2.2. Fuzzy Linear programming

Fuzzy Linear programming (FLP) is a methodology used to solve optimization problems by integrating uncertainty in linear program models. The solution with FLP is the search for a value Z which is an objective function to be optimized in such a way that it is subject to constraints that are modeled using fuzzy sets [3]. The general form of the FLP model can be seen in Equation below: [4]

\[
Z = \sum_{j=1}^{n} \tilde{c}_j x_j
\]

(1)

With constraints \( \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i, i = 1, ..., m ; m, n \in N \).

The FLP problem with fuzzy technical coefficients is presented by determining the constraints and objective functions to be achieved from the decision variables in the form of linear inequalities [4]

\[
z = \sum_{j=1}^{n} c_j x_j,
\]

(2)

with constraints \( \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i, 1 \leq i \leq m, m \in N \)

\( x_j \geq 0, \quad 1 \leq j \leq n, n \in N \)

Description:

\( x_j \): decision variable

\( \tilde{a}_{ij} \): technical coefficient in fuzzy number form

\( \tilde{b}_i \): right segment coefficient in fuzzy number form

\( \tilde{c}_j \): cost coefficient in fuzzy number form

2.3. Linear Fuzzy Programming with Right-hand Constants as Fuzzy Numbers

Fuzzy Linear programming (FLP) is the search for a value z which is an objective function that will be optimized in such a way that it is subject to constraints that are modeled using fuzzy sets [5].

The assumption that the linear program decision will be made in a fuzzy environment, will change slightly, i.e. the imperative form of the objective function is not imperative:
a. The imperative form of the objective function is no longer strictly “maximum” or “minimum” because there are several things that need to be considered in a system.
b. The ≤ sign (on the limit) in the maximization case and the ≥ sign (on the limit) in the minimization case are no longer mathematically crisp, but have a slight violation of meaning. This is also due to the fact that there are several considerations in the system that cause the constraints to not be approached unequivocally [3].

Here are some FLP models.

a. FLP with the right-hand constant value ($b_i$) is a fuzzy number.
b. FLP where the right-hand constant ($b_i$) and the $a_{ij}$ coefficients of the constraint matrix are fuzzy numbers.
c. FLP with objective function coefficient ($c_i$) is a fuzzy number.
d. FLPs with variables are fuzzy numbers.

The following explanation will only discuss part a of the FLP model, which is the FLP where the right-hand constant ($B_i$) is a fuzzy number. The form of the linear program in this case is as follows [6].

Maximize/Minimize $\sum_{j=1}^{n} \hat{c}_j x_j$

Conditional on

$$\sum_{j=1}^{n} \hat{a}_{ij} x_j \leq \hat{b}_i$$

$$x_j \geq 0$$

The membership value of the fuzzy number $\hat{B}_i$ is

$$\hat{B}_i = \begin{cases} 1; & \text{when } x \leq \hat{b}_i - \frac{\hat{b}_i + p_i - x}{p_i}; \\ \text{when } \hat{b}_i < x < \hat{b}_i + p_i; & \text{0; when } \hat{b}_i + p_i \leq x \end{cases}$$

$G$ is the optimal value of the fuzzy set, which is a fuzzy subset of $\mathbb{R}^n$, defined as follows

$$G(x) = \begin{cases} 1; & \text{when } z_u \leq cx - \frac{cx + z_l - x}{z_u - z_l}; \\ \text{when } z_l < cx < z_u; & \text{0; when } cx \leq z_l \end{cases}$$

Next, optimize the problem by maximizing or minimizing $\lambda$.

Maximize/Minimize $\lambda$

With the condition that

$$\lambda(z_u - z_l) - cx \leq -z_l$$

$$\lambda(p_i) + \sum_{i=0}^{n} a_{ij} x_j \leq b_i + p_i \ (i \in N_m)$$

$$\lambda, x_j \geq 0 \ (j \in N_n)$$

2. 4. Simplex Method

One of the optimal solutions used in linear programming is the simplex method. The steps of the simplex method are as follows: [7]

1. Changing the objective function and objective function constraints are converted into implicit functions, meaning that all $c_j x_j$ are shifted to the left.
2. Arrange the equations in the simplex table.
3. Selecting the key column. A key column is a column that is the basis for changing the initial table. The column that has the largest negative value in the objective function row is chosen. If a table no longer has an negative value in the objective function row, it cannot be optimized.
4. Selecting the key row. The key row is the row that is the basis for transforming the table above. For that, first find the index of each row by dividing the values in the NK column by the inline values in the key column.
Index = \frac{NK \text{ column value}}{key \text{ column value}}

Then the number with the smallest positive value is selected.
5. Changing the key row values. The key row values are changed by dividing them by the key number.
6. Change the values other than the key row. The other row values other than the key row can be changed with the following formula:
   \[\text{new row} = \frac{\text{old row}}{\text{key column coefficient}} \times \text{new key row value}\]
7. Continue the improvements or changes. Repeat the improvement steps from step 3 to step 6 to improve the tables that have been changed or improved in value. The changes stop after the first row (objective function) has no negative values.

2. 5. Place and Time of Research

This research was carried out by collecting data directly through interviews with the factory owner located in Gg. Rotansia, Tj. Sari, Kec. Medan Selayang, Medan City, North Sumatra on March 16, 2024.

2. 6. Research Approach

The approach used in this research is a quantitative approach. Quantitative approach is a research method that uses numbers and statistics in data collection and data analysis that can be measured. The method used in this research is a direct interview with the factory owner. This interview process will be an instrument in collecting all data used in this research.

2. 7. Data Analysis Technique

In quantitative research, data analysis techniques are methods used to process data and draw conclusions from the amount of data that has been collected. In this research, the data is processed using the simplex method by utilizing POM-QM software to help complete complex calculations.

3. RESULT AND DISCUSSION

The Untir-untir and Raja Manis Cookie Factory produces two types of cookies, Untir-untir and Raja Manis, using raw materials such as wheat flour, sugar, yeast, salt, baking powder, sesame seeds, butter, and starch. The profit earned per kg of product for both Untir-untir and Raja Manis is Rp.13,500/kg. The raw material requirements to produce 1 kg of each type of bread in kg are listed in Table 1.

<table>
<thead>
<tr>
<th>Raw Material (kg)</th>
<th>Untir-untir</th>
<th>Raja Manis</th>
<th>Inventory (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tepung Terigu</td>
<td>0.7</td>
<td>0.65</td>
<td>100</td>
</tr>
<tr>
<td>Gula Pasir</td>
<td>0.17</td>
<td>0.165</td>
<td>25</td>
</tr>
<tr>
<td>Ragi</td>
<td>0.013</td>
<td>0.01</td>
<td>2</td>
</tr>
<tr>
<td>Garam</td>
<td>0.021</td>
<td>0.02</td>
<td>3.5</td>
</tr>
<tr>
<td>Baking Powder</td>
<td>0.01</td>
<td>0.08</td>
<td>1.6</td>
</tr>
<tr>
<td>Wijen</td>
<td>0</td>
<td>0.13</td>
<td>4</td>
</tr>
<tr>
<td>Mentega</td>
<td>0.03</td>
<td>0.25</td>
<td>4</td>
</tr>
<tr>
<td>Tepung Kanji</td>
<td>0.01</td>
<td>0.005</td>
<td>1</td>
</tr>
</tbody>
</table>

But it turns out that sometimes the amount of raw material inventory available at the Untir-untir and Raja Manis factories is uncertain. To get maximum profit, the problem cannot be solved using the usual Linear programming (LP) model. So, to solve it can use the Fuzzy Linear programming (FLP) model with the right-hand constant coefficient is a fuzzy number.
In this study, researchers increased the inventory of each raw material by 10%, then compared it with no additional raw materials.

Based on Table 1, the decision variables are:

\[ x_1 : \text{Number of Untir-untir produced} \]
\[ x_2 : \text{Number of Raja Manis produced} \]

So, the formulation of the mathematical model is as follows:

Maximize: 
\[ z = 13.500x_1 + 13.500x_2 \]

With constraints:
\[ 0.7x_1 + 0.65x_2 \leq 100 \]
\[ 0.17x_1 + 0.165x_2 \leq 25 \]
\[ 0.013x_1 + 0.01x_2 \leq 2 \]
\[ 0.021x_1 + 0.02x_2 \leq 3.5 \]
\[ 0.01x_1 + 0.08x_2 \leq 1.6 \]
\[ 0x_1 + 0.13x_2 \leq 4 \]
\[ 0.03x_1 + 0.25x_2 \leq 4 \]
\[ 0.01x_1 + 0.005x_2 \leq 1 \]
\[ x_1, x_2 \geq 0 \]

By using POM-QM software, the solution is obtained 
\[ z = 1378.723, x_1 = 97.87, \text{ and } x_2 = 4.25 \]

If there is an increase in the amount of inventory, the mathematical model is as follows:

Max \[ z = 13.500x_1 + 13.500x_2 \]

With constraints:
\[ 0.7x_1 + 0.65x_2 \leq 100 + p_1 \]
\[ 0.17x_1 + 0.165x_2 \leq 25 + p_2 \]
\[ 0.013x_1 + 0.01x_2 \leq 2 + p_3 \]
\[ 0.021x_1 + 0.02x_2 \leq 3.5 + p_4 \]
\[ 0.01x_1 + 0.08x_2 \leq 1.6 + p_5 \]
\[ 0x_1 + 0.13x_2 \leq 4 + p_6 \]
0.03x_1 + 0.25x_2 \leq 4 + p_7
0.01x_1 + 0.005x_2 \leq 1 + p_8
x_1, x_1 \geq 0

The linear program problem with the above model can be solved with Fuzzy Linear programming (FLP) with the right-hand coefficient being a fuzzy number.

For an increase in the amount of inventory by 10%, the mathematical model becomes:
Max \ z = 13.500x_1 + 13.500x_2
With constraints:
0.7x_1 + 0.65x_2 \leq 100 + 100(0.1)t 
0.17x_1 + 0.165x_2 \leq 25 + 25(0.1)t 
0.013x_1 + 0.01x_2 \leq 2 + 2(0.1)t 
0.021x_1 + 0.02x_2 \leq 3.5 + 3.5(0.1)t 
0.01x_1 + 0.08x_2 \leq 1.6 + 1.6(0.1)t 
x_1 + 0.13x_2 \leq 4 + 4(0.1)t 
0.03x_1 + 0.25x_2 \leq 4 + 4(0.1)t 
0.01x_1 + 0.005x_2 \leq 1 + 1(0.1)t 
x_1, x_1 \geq 0

Two forms of optimization will be performed, namely \ t = 0 \ and \ t = 1.
- For \ t = 0, \ the linear program form and solution are obtained as shown in the calculation results using ordinary Linear Programming.
- For \ t = 1, \ the following linear program form is obtained.
Max \ z = 13.500x_1 + 13.500x_2
With constraints:
0.7x_1 + 0.65x_2 \leq 110 
0.17x_1 + 0.165x_2 \leq 27.5 
0.013x_1 + 0.01x_2 \leq 2.2 
0.021x_1 + 0.02x_2 \leq 3.85 
0.01x_1 + 0.08x_2 \leq 1.96 
x_1 + 0.13x_2 \leq 4.4 
0.03x_1 + 0.25x_2 \leq 4.4 
0.01x_1 + 0.005x_2 \leq 1.1 
x_1, x_1 \geq 0

By using POM-QM software, the solution is obtained \ z = 1.516.596, \ x_1 = 107,65 dan \ x_2 = 4,68.

![Figure3. Input Display in POM-QM Software for FLP Solution (t = 1)](image-url)
From these two results, the value of $P_0$ can be determined, can be determined, which is the difference between $z$ at time $t = 1$ and $z$ at time $t = 0$. $P_0$ serves for FLP formation. Obtained $P_0 = 1.516.596 - 1.378.723 = 137.873$. Next, the completion will be done with the FLP.

<table>
<thead>
<tr>
<th>Fuzzy Boundaries</th>
<th>$t = 0$</th>
<th>$t = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective Function</td>
<td>1.378.723</td>
<td>1.516.596</td>
</tr>
<tr>
<td>Constraint 1</td>
<td>97.87</td>
<td>107.65</td>
</tr>
<tr>
<td>Constraint 2</td>
<td>4.25</td>
<td>4.68</td>
</tr>
</tbody>
</table>

With constraints $\lambda = 1 - t$, FLP model can be formed as follows.

Maximize $\lambda$

With constraints

$137.873\lambda - 13.500x_1 - 13.500x_2 \leq -1.516.596 + 137.873 = -1.378.723$

$10\lambda + 0.7x_1 + 0.65x_2 \leq 100 + 10 = 110$

$2.5\lambda + 0.17x_1 + 0.165x_2 \leq 25 + 2.5 = 27.5$

$0.2\lambda + 0.013x_1 + 0.01x_2 \leq 2 + 0.2 = 2.2$

$0.35\lambda + 0.021x_1 + 0.02x_2 \leq 3.5 + 0.35 = 3.85$

$0.16\lambda + 0.01x_1 + 0.08x_2 \leq 1.6 + 0.16 = 1.76$

$0.4\lambda + 0.13x_1 + 0.13x_2 \leq 4 + 0.4 = 4.4$

$0.4\lambda + 0.03x_1 + 0.25x_2 \leq 4 + 0.4 = 4.4$

$0.1\lambda + 0.01x_1 + 0.005x_2 \leq 1 + 0.1 = 1.1$

$\lambda, x_1, x_1 \geq 0$

So, the form of the linear program becomes

$-137.873\lambda + 13.500x_1 + 13.500x_2 \geq 1.378.723$

$10\lambda + 0.7x_1 + 0.65x_2 \leq 110$

$2.5\lambda + 0.17x_1 + 0.165x_2 \leq 27.5$

$0.2\lambda + 0.013x_1 + 0.01x_2 \leq 2.2$

$0.35\lambda + 0.021x_1 + 0.02x_2 \leq 3.85$

$0.16\lambda + 0.01x_1 + 0.08x_2 \leq 1.76$

$0.4\lambda + 0.13x_1 + 0.13x_2 \leq 4.4$

$0.4\lambda + 0.03x_1 + 0.25x_2 \leq 4.4$

$0.1\lambda + 0.01x_1 + 0.005x_2 \leq 1.1$

$\lambda, x_1, x_1 \geq 0$

By using POM-QM software, the solution is obtained $\lambda = 0.5, x_1 = 102.76$ dan $x_2 = 4.46$
Figure 5. Input Display in POM-QM Software for FLP Solution

Figure 6. Solution display for FLP \((t = 1)\) in POM-QM Software

So, the \(z\) value obtained is
\[
z = 13.500x_1 + 13.500x_2
\]
\[
z = 13.500(102.76) + 13.500(4.46)
\]
\[
z = 1.447.470
\]

With the values for each constraint being
\[
0.7x_1 + 0.65x_2 = (0.7)(102.76) + (0.65)(4.46) = 74.83
\]
\[
0.17x_1 + 0.165x_2 = (0.17)(102.76) + (0.165)(4.46) = 18.2
\]
\[
0.013x_1 + 0.01x_2 = (0.013)(102.76) + (0.01)(4.46) = 1.38
\]
\[
0.021x_1 + 0.02x_2 = (0.021)(102.76) + (0.02)(4.46) = 2.24
\]
\[
0.01x_1 + 0.08x_2 = (0.01)(102.76) + (0.08)(4.46) = 1.38
\]
\[
0x_1 + 0.13x_2 = (0)(102.76) + (0.13)(4.46) = 0.57
\]
\[
0.03x_1 + 0.25x_2 = (0.03)(102.76) + (0.25)(4.46) = 4.19
\]
\[
0.01x_1 + 0.005x_2 = (0.01)(102.76) + (0.005)(4.46) = 1.04
\]

Thus, in order to obtain a maximum profit of IDR 1,447,470, 102.76 kg of Untir-untir and 4.46 kg of Raja Manis should be produced; with the addition of 0.19 ounces of butter; 0.04 ounces of starch; and no addition of other raw materials.

<table>
<thead>
<tr>
<th>Table 3. Comparison of LP and FLP solutions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product Type</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Untir-untir ((x_1))</td>
</tr>
<tr>
<td>Raja Manis ((x_2))</td>
</tr>
<tr>
<td>Amount</td>
</tr>
<tr>
<td>Profit</td>
</tr>
</tbody>
</table>
4. CONCLUSION

Based on the discussion, the production optimization of small and medium micro enterprises Untir-untir and Raja Manis Factory can not only be solved by Linear Programming, but also by Fuzzy Linear Programming. For the case of small and medium micro enterprises, the FLP model is used with the right-hand constant value ($b_i$) is a fuzzy number. Using LP, a profit of Rp. 1,378,723 was obtained by producing 97.87 kg of Untir-untir and 4.25 kg of Raja Manis. While using FLP obtained a profit of Rp.1,447,470 by producing 102.76 kg of Untir-untir and 4.46 kg of Raja Manis and by adding 0.19 ounces of butter, 0.04 ounces of starch, and no addition to other raw materials.

REFERENCES