Journal of Research in Mathematics Trends and Technology
Journal homepage: https://talenta.usu.ac.id/jormtt

Optimization of Production of Cendol Dawet Ice and Doger Ice on Pakde Carts Using Fuzzy Linear Programming

T.J. Marpaung [1*], Apredo Suranta Singarimbun [2], Wike Joesline Lumban Tobing [2], Yessi Mardina Manik [2], Ismayadi [3]

[1*] Statistics Department, University of North Sumatra, Medan, 20155, Indonesia
[2] Mathematics Department, University of North Sumatra, Medan, 20155, Indonesia
[3] Department of Community Nursing, Universitas Sumatera Utara, 20155, Medan, Indonesia

[*] Corresponding Author: tj.marpaung@usu.ac.id

ARTICLE INFO

Article history:
Received: 08 January 2024
Revised: 09 February 2024
Accepted: 29 March 2024
Available online: 30 March 2024

E-ISSN: 2656-1514
P-ISSN:

How to cite:
Marpaung, T.J., Singarimbun, A., Tobing, W., Manik, Y, "Optimizing the Production of Cendol Dawet Ice and Doger Ice on Pakde Carts Using *Fuzzy Linear Programming*", Journal of Research in Mathematics Trends and Technology, vol. V6, no. 1, Mar. 2024, doi: 10.32734/jormtt.v6i1.16699

ABSTRACT

Fuzzy Linear Programming is a model that is often used to solve problems related to optimization problems. This research will discuss efforts to optimize the production of Es cendol dawet and Es doger drinks on PakDe's cart in order to obtain maximum profits with minimum expenditure, without reducing the quality of the goods produced. One method used to obtain maximum benefits is to model goals and constraints using linear programming. Solving fuzzy linear programming is assisted using the QM for Windows application. Calculation results using fuzzy linear programming, maximum income will be obtained if 45 servings of Es Cendol Dawet are produced and 45 servings of Es Doger are produced, so the profit obtained is IDR 450,000. The increase in profits is IDR 50,000 more than ordinary linear programming.

Keywords: Optimization fuzzy logic, linear programming, fuzzy linear programming, POM-QM

1. INTRODUCTION

Optimization is a process or technique for making something as good as possible or achieving optimal results in a particular context [1]. In the mathematical discipline, optimization refers to the study of problems that try to find the minimum or maximum value of a real function. According to Suprodjo and Purwandi, 1982 in Tarmizi, 2005, mathematically optimization is a way of getting extreme values, either maximum or minimum, of a certain function with its limiting factors. One way to solve optimization problems is linear programming [2].

Linear programming is a mathematical model in which the objective function and constraints can be expressed as linear functions of decision variables that can maximize profits or productivity, or minimize costs or consumption. [3]. A linear programming problem can be solved using the graphic method or the Simplex method, for two or more variables the Simplex method can be used. Parameters in a linear program are data consisting of objective function coefficients, constants and Right-Hand-Side coefficients which are known with certainty. Because these parameters are known with certainty, the assumptions in the linear programming problem are assumptions of certainty. However, in real life these assumptions are rarely met and uncertainty often occurs. So a method emerged that can be used to solve vague problems for production planning, namely Fuzzy Linear Programming.

In this era of globalization, culinary businesses such as Gerobak Pakde play an important role in maintaining Indonesia's rich culture through its traditional dishes, including Es Cendol Dawet and Es Doger. However, to remain competitive and develop amidst increasingly fierce competition, an efficient strategy is needed in managing raw materials, production processes and determining production quantities. Uncertainty in daily or seasonal demand as well as fluctuations in raw material prices can be challenges that must be overcome.

Through the application of Fuzzy Linear Programming, Gerobak Pakde can optimize the production of Es Cendol Dawet and Es Doger by taking into account various important factors such as market demand, availability of raw materials, production capacity and customer preferences. Previously there had been research that applied the fuzzy goal programming method which was completed using the simplex method [4]. Thus, the aim of this research is to optimize production and maximize profits.

2. RESEARCH METHODS

The research carried out was applied research on the Es Cendol Dawet and Es Doger Gerobak Pakde businesses. Data collection techniques include conducting interviews with business owners. The data collected in the research are data on raw materials used, data on raw material prices, product selling prices, and production quantities. Data processing uses *software assistance* , namely *POM-QM* .

The steps in data processing are as follows [5]:

1. Data retrieval
   The data that will be used in this research is data from March 2024 through interviews at Gerobak Pakde.
2. Creating *Linear Programming Models*
   Create a model based on the data that has been taken, namely determining the objective function and constraint function. In making this model, the product types Es Cendol Dawet and Es Doger will be variables $x_1$and $x_2$will be maximized as the objective function. Meanwhile, raw materials will be a constraint function.
3. Adding Tolerance Values to the Model
   The addition of tolerance values to the model is intended as a *fuzzy limit.*
4. *Linear Programming* Models
   The model that has been created will be completed using *POM-QM software.*
5. *Fuzzy* Boundaries
   This is the value obtained from solving the *linear programming model.*
6. *Fuzzy Linear Programming* Model
   fuzzy linear programming model is formed based on existing constraint functions and added with *fuzzy constraint values.*
7. *Fuzzy Linear Programming* Models
   The solution to *fuzzy linear programming* is carried out using *POM-QM software.*

2.1 Fuzzy Logic

Fuzzy is linguistically defined as hazy or vague [6]. In *fuzzy*, there is a degree of membership that has a value range of 0 (zero) to 1 (one). *Fuzzy* logic is a logic that has a value of vagueness or vagueness *between* true or false. In *fuzzy* logic theory A value can, be true or false at the same time. However, the extent of truth and falsehood depends on the membership weight it has. *Fuzzy* Logic has a degree of membership in the range 0 to 1. Several reasons why people use Fuzzy Logic include [6]:

1. The concept of Fuzzy Logic is easy to understand. The mathematical concepts underlying fuzzy reasoning are very simple and easy to understand.
2. Fuzzy logic is very flexible, meaning it is able to adapt to changes and uncertainties that accompany problems.
3. Fuzzy logic has tolerance for inaccurate data.
4. Fuzzy logic is able to model very complex nonlinear functions.
5. Fuzzy Logic can build and apply the experiences of experts directly without having to go through a training process.
6. Fuzzy logic can work together with conventional control techniques.
7. Fuzzy Logic uses everyday language so it is easy to understand.

2.2 Linear programming

Linear programming is a method used to solve problems related to linear optimization. *Linear programming* is used to determine the maximum or minimum value of an objective function, while paying attention to existing constraints. *Linear programming* models has three basic components, namely: decision variables to be determined, objectives or goals that need to be optimized (maximized or minimized) and constraints that must be resolved.

The mathematical model of *Linear Programming* is as follows [7]:

$$Z = \sum_{j=1}^{n} c_j x_j \tag{1}$$

With constraints:

$$\sum_{j=1}^{n} a_{ij} x_j \ (\leq, =, \geq) b_i \tag{2}$$

For all values i $(i = 1,2,\ldots,m)$.

Where :

Z= optimized objective function value (maximum or minimum)

x_j = jth decision variable $(j = 1,2,\ldots,n)$

a_{ij}= technological coefficient

c_j= objective function coefficient

b_i= right side coefficient

2.3 Fuzzy Linear Programming

Fuzzy Linear Programming is searching the value of the objective function that will be optimized, with certain constraints modeled using fuzzy sets. For example, $p_i \geq 0$is the tolerance interval that is allowed to be violated both from the objective function and the constraints of b_i show there is a fuzzy element, then formulation the mathematical model is [7]

$$Z = \sum_{j=1}^{n} c_j x_j \tag{3}$$

With constraints:

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i + p_i \tag{4}$$

$x_j \geq 0,$ with $i = 1,2,\ldots,m$ And $j = 1,2,\ldots,n$

fuzzyfication process in Equation 1 and Equation 2 which has been completed will produce a solution in the form of Z^0(*lowerbound*) where this process does not use tolerance values. Next, equation 3 and equation 4 have been solved will produce Z^1(*upper bound*) where this process uses a tolerance value. The following is defined as a membership function that uses a trapezium curve representation of the i -th constraint function , which is shown in Figure 1.

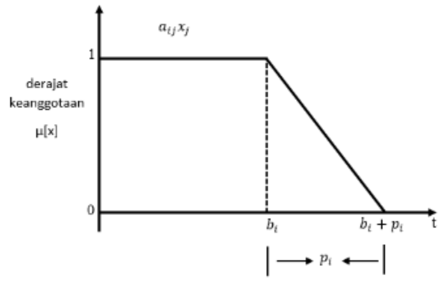


Figure 1. Membership Function

$$\mu_i\left(\sum_{j=1}^{n} a_{ij}\, x_j\right) = \begin{cases} 1 & ; \sum_{j=1}^{n} a_{ij}\, x_j < b_i \\ 1 - \dfrac{\sum_{j=1}^{n} a_{ij}\, x_j - b_i}{p_i} & ; \sum_{j=1}^{n} a_{ij}\, x_j < b_i + p_i \\ 0 & ; \sum_{j=1}^{n} a_{ij}\, x_j < b_i + p_i \end{cases} \tag{5}$$

With

$b_i + p_i$= right side of the ith constraint

Defuzzification process carried out after getting *the lower bound value* and *upper bounds.* This process will be forming a new line program and to complete it a 2-phase method is used. Equality from *fuzzy linear program m ing* that is:

Maximize:λ

With constraints:

$$\lambda \leq \mu_0(x) \text{ atau } \sum_{j=1}^{n} c_j x_j - \lambda\,(Z^1 - Z^0) \geq Z^0 \tag{6}$$

$$\lambda \leq \mu_1(x) \text{ atau } \sum_{j=1}^{n} a_{ij} x_j - \lambda\,(p_i) \geq b_i + p_i \tag{7}$$

$$p > 0; \lambda \in [0,1];\ \text{dan } x_j \geq 0$$

With$i = 1,2,\dots,m$ dan $j = 1,2,\dots,n$

3. RESULTS AND DISCUSSION

1. Data collection

Based on the data that has been obtained, a production table is formed as follows:

Table 1. Production raw material data

Raw material	Types of products		Production Needs		Unit
	Dawet Cendol Ice	Doger ice	Amount of Raw Materials	Tolerance	
Coconut cream	100	0	4000	1000	ml
Milk	0	12	500	125	gr
Doger Water	0	150	6000	1500	ml
Cendol	50	0	2000	500	gr
Brown Sugar Water	100	0	4000	1000	ml
Crystal Ice	150	150	12000	3000	gr
Profit	5	5			Thousand rupiah

2. Model Formulation

- Decision Variables

  The decision variable is the variable for which the activity level value will be sought based on existing resources. Based on the data above, the decision variables for the problem are as follows:

  x_1 =the number of Es Cendol Dawet produced

  x_2 =the number of Es Doger produced

- Constraint Function

  The constraint function is a form of inequality or equation function that states the number of activity levels that are limited by the amount of available resources. From the production raw material data table, the constraint function is:

$$
\begin{gathered}
100x_1 \leq 4000 + 1000t \\
12x_2 \leq 500 + 125t \\
150x_2 \leq 6000 + 1500t \\
50x_1 \leq 2000 + 500t \\
100x_1 \leq 4000 + 1000t \\
150x_1 + 150x_2 \leq 12000 + 3000t \\
x_1, x_2 \geq 0
\end{gathered}
$$

- Objective Function

  The objective function is a function whose maximum or minimum value will be determined. The value of the objective function itself depends on the contribution per unit for each decision variable. Based on the data above, the form of the objective function is

  Maximize:$Z = 5x_1 + 5x_2$

  Furthermore, based on the data above, the complete formulation is:

  Maximize:$Z = 5x_1 + 5x_2$

  With Limitations:

$$
\begin{gathered}
100x_1 \leq 4000 + 1000t \\
12x_2 \leq 500 + 125t \\
150x_2 \leq 6000 + 1500t \\
50x_1 \leq 2000 + 500t \\
100x_1 \leq 4000 + 1000t \\
150x_1 + 150x_2 \leq 12000 + 3000t \\
x_1, x_2 \geq 0
\end{gathered}
$$

3. Solving with *Linear Programming*

Next, using the simplex method which is calculated using *software POM-QM* , *fuzzy* boundary values will be searched based on the problem formulation that has been created. From this formulation, maximum profits will be sought.

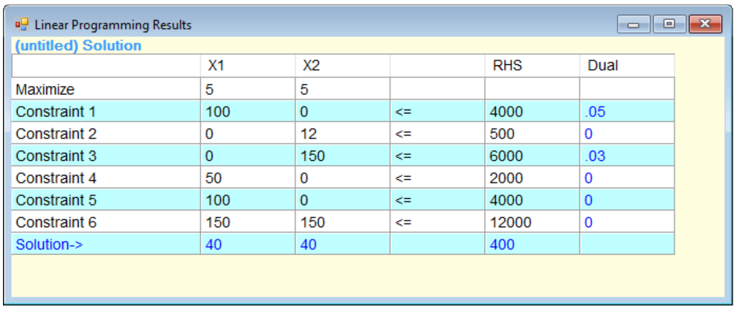
- For $t = 0$ $(\lambda = 1)$, the model is obtained:

  Maximize:$Z = 5x_1 + 5x_2$

  With Limitations:

$$
\begin{gathered}
100x_1 \leq 4000 \\
12x_2 \leq 500 \\
150x_2 \leq 6000 \\
50x_1 \leq 2000 \\
100x_1 \leq 4000 \\
150x_1 + 150x_2 \leq 12000 \\
x_1, x_2 \geq 0
\end{gathered}
$$

By using *software POM-QM* , the following solution is obtained:
$x_1 = 40$
$x_2 = 40$
$Z = 400$


Linear Programming Results
(untitled) Solution

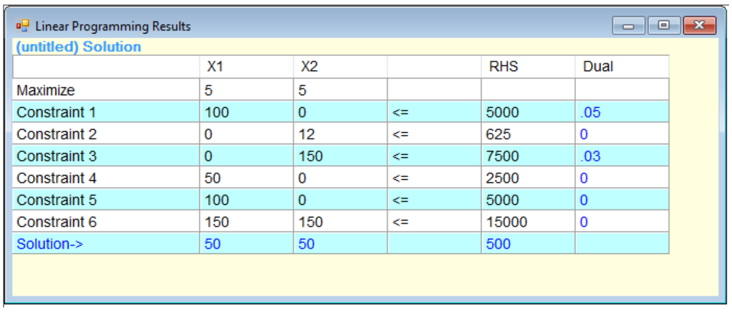
	X1	X2		RHS	Dual
Maximize	5	5			
Constraint 1	100	0	<=	4000	.05
Constraint 2	0	12	<=	500	0
Constraint 3	0	150	<=	6000	.03
Constraint 4	50	0	<=	2000	0
Constraint 5	100	0	<=	4000	0
Constraint 6	150	150	<=	12000	0
Solution->	40	40		400	

Figure 2. *Linear Programming* Model *Output* ($t = 0,\ \lambda = 1$)

- For $t = 1$ $(\lambda = 0)$, the model is obtained:
  Maximize:$Z = 5x_1 + 5x_2$
  With Limitations:

$$100x_1 \leq 5000$$
$$12x_2 \leq 625$$
$$150x_2 \leq 7500$$
$$50x_1 \leq 2500$$
$$100x_1 \leq 5000$$
$$150x_1 + 150x_2 \leq 15000$$
$$x_1, x_2 \geq 0$$

By using *software POM-QM* , the following solution is obtained:
$x_1 = 50$
$x_2 = 50$
$Z = 500$


Linear Programming Results
(untitled) Solution

	X1	X2		RHS	Dual
Maximize	5	5			
Constraint 1	100	0	<=	5000	.05
Constraint 2	0	12	<=	625	0
Constraint 3	0	150	<=	7500	.03
Constraint 4	50	0	<=	2500	0
Constraint 5	100	0	<=	5000	0
Constraint 6	150	150	<=	15000	0
Solution->	50	50		500	

Figure 3. *Linear Programming* Model *Output* ($t = 1,\ \lambda = 0$)

From these two results, the value can be determined P_0, namely the difference between Zat time $t = 1$and Zat time $t = 0$. P_0functions for the formation of *fuzzy linear programming*. Obtained $P_0 = 500 - 400 = 100$.

4. Solution with *Fuzzy Linear Programming*

the following *fuzzy* constraints are obtained:

Table 2. *Fuzzy* Constraints

	Fuzzy Boundaries	
	$t = 0$	$t = 1$
Objective Function	400	500
Limitation 1	4000	5000
Limitation 2	500	625

Limitation 3	6000	7500
Limitation 4	2000	2500
Limitation 5	4000	5000
Limitation 6	12000	15000

With these limitations $\lambda = 1 - t$, finally a *fuzzy linear programming model can be formed* , namely:
Maximize:$Z = \lambda$
With Limitations:

$$\begin{aligned} 100\lambda - 5x_1 - 5x_2 &\leq -500 + 100 = -400 \\ 1000\lambda + 100x_1 &\leq 4000 + 1000 = 5000 \\ 125\lambda + 12x_2 &\leq 500 + 125 = 625 \\ 1500\lambda + 150x_2 &\leq 6000 + 1500 = 7500 \\ 500\lambda + 50x_1 &\leq 2000 + 500 = 2500 \\ 1000\lambda + 100x_1 &\leq 4000 + 1000 = 5000 \\ 3000\lambda + 150x_1 + 150x_2 &\leq 12000 + 3000 = 15000 \\ \lambda, x_1, x_2 &\geq 0 \end{aligned}$$

So that the right side of the above formulation does not have a negative value, the *linear programming form formulation* above is changed to:
Maximize:$Z = \lambda$
With Limitations:

$$\begin{aligned} -100\lambda + 5x_1 + 5x_2 &\geq 400 \\ 1000\lambda + 100x_1 &\leq 5000 \\ 125\lambda + 12x_2 &\leq 625 \\ 1500\lambda + 150x_2 &\leq 7500 \\ 500\lambda + 50x_1 &\leq 2500 \\ 1000\lambda + 100x_1 &\leq 5000 \\ 3000\lambda + 150x_1 + 150x_2 &\leq 15000 \\ \lambda, x_1, x_2 &\geq 0 \end{aligned}$$

By using POM-QM *software* , the following solution is obtained:
$\lambda = 0{,}5$
$x_1 = 45$
$x_2 = 45$
$Z = 450$

Linear Programming Results
(untitled) Solution

	λ	X1	X2		RHS	Dual
Maximize	1	0	0			
Constraint 1	-100	5	5	>=	400	-.01
Constraint 2	1000	100	0	<=	5000	0
Constraint 3	125	0	12	<=	625	0
Constraint 4	1500	0	150	<=	7500	0
Constraint 5	500	50	0	<=	2500	0
Constraint 6	1000	100	0	<=	5000	0
Constraint 7	3000	150	150	<=	15000	0
Solution->	.5	45	45		.5	

Figure 3. *Fuzzy* Model *Output Linear Programming*

The values for each constraint are:
- Limitation 1= $100x_1 = (100)(45) = 4500$
- Limitation 2= $12x_2 = (12)(45) = 540$
- Limitation 3= $150x_2 = (150)(45) = 6750$
- Limitation 4= $50x_1 = (50)(45) = 2250$
- Limitation 5= $100x_1 = (100)(45) = 4500$
- Limitation 6= $150x_1 + 150x_2 = (150)(45) + (150)(45) = 13500$

Based on the results above, the solutions for the *fuzzy linear programming model* and the ordinary linear programming model can be seen in the following table:

Table 3. *Non-Fuzzy* and *Fuzzy* Solutions

Variable	*Fuzzy* Solution	*Fuzzy* Solution
x_1	40	45
x_2	40	45
Z	400	450
Raw material	Boundary Value	
Coconut cream	4000	4500
Milk	500	540
Doger Water	6000	6750
Cendol	2000	2250
Brown Sugar Water	4000	4500
Crystal Ice	12000	13500
$\lambda - cut$		0.5

When using *fuzzy linear programming,* maximum income will be obtained if as many 45packs of Es Cendol Dawet are produced and as many 45packs of Es Doger are produced, the profit obtained (Z) is IDR. 450,000 (Rp. 50,000 more than regular *linear programming)*. Note that in this condition, the raw materials required are coconut milk in the amount of 4500ml, milk in the amount of 540grams, doger water in the amount of 6750ml, cendol in the amount of 2250grams, brown sugar water in the amount of 4500ml, and crystal ice in the amount of 13500grams. These results require the addition of coconut milk as much as 500ml from the 4000 ml provided, milk as much as 40grams from the 500 grams provided, doger water as much as 750ml from the 6000 ml provided, cendol as much as 250grams from the 2000 grams provided, brown sugar water as much as 500ml from the 4000 ml provided. ml provided, and 1500grams of ice crystals from the 12000 grams provided.

4. CONCLUSION

By using *fuzzy linear programming,* maximum income will be obtained if 45 packs of Es Cendol Dawet are produced and 45 packs of Es Doger are produced, so the profit obtained is IDR. 450,000 (Rp. 50,000 more than regular *linear programming)*.

BIBLIOGRAPHY

[1] SS Rao, *Engineering optimization: theory and practice*. John Wiley & Sons, 2019.

[2] L. Programing and BT Ujito, "OPTIMIZATION OF HOUSE TYPE SELECTION USING LINEAR PROGRAMING TECHNIQUES CASE STUDY: PONDOK SUKUN CLUSTER MALANG CITY".

[3] M. Carter, CC Price, and G. Rabadi, *Operations research: a practical introduction* . Chapman and Hall/CRC, 2018.

[4] C. Alfiani, M. Zavina, U. Khasanah, MN Fadli, and A. Indahsari, "APPLICATION OF FUZZY GOAL PROGRAMMING IN OPTIMIZING PRODUCTION PLANNING," *Lebesgue Journal: Scientific Journal of Mathematics, Mathematics and Statistics Education* , vol. 3, no. 2, pp. 387–396, 2022.

[5] MI PuluANGKA, Y. Langi, and AJ Rindengan, "Optimization of production planning at CV. Gorontalo Real Work Furniture using a fuzzy linear programming model," *d'CARTESIAN: Journal of Mathematics and Applications* , vol. 7, no. 2, pp. 78–83, 2018.

[6] S. Kusumadewi and H. Purnomo, "Application of Fuzzy Logic for decision support," *Yogyakarta: Graha Ilmu* , vol. 2, 2010.

[7] HA Taha, "Operations research an introduction," 2007.