Journal of Research in Mathematics Trends and Technology
Journal homepage: https://talenta.usu.ac.id/jormtt

Efficiency Of Environmental Performance Measurement Using Data Envelopment Analysis (DEA) With Fuzzy Approach

Nurul Nadhila[1], Tulus*[2], Mardiningsih[3]
[1]*Department of Mathematics, Universitas Sumatera Utara, Medan, 20155, Indonesia*
[2]*Department of Mathematics, Universitas Sumatera Utara, Medan, 20155, Indonesia*
[3]*Department of Mathematics, Universitas Sumatera Utara, Medan, 20155, Indonesia*
*Corresponding Author: tulus@usu.ac.id

ARTICLE INFO

Article history:
Received: 03 July 2025
Revised: 01 August 2025
Accepted: 13 September 2025
Available online: 19 September 2025

E-ISSN: 2656-1514
P-ISSN:

How to cite:
Nadhila, N., Tulus, Mardiningsih, "Efficiency Of Environmental Performance Measurement Using Data Envelopment Analysis (DEA) With Fuzzy Approach", Journal of Research in Mathematics Trends and Technology, vol. V7, No. 2, September. 2025, doi: 10.32734/jormtt.v7i2.21717

ABSTRACT

This study aims to analyze and measure environmental performance efficiency in residential areas in Medan City using the Data Envelopment Analysis (DEA) method combined with a fuzzy approach. Environmental performance is measured based on relevant inputs and outputs, where the data used are secondary data obtained from previous research reports. Fuzzification is applied to address uncertainty in the data by converting input and output values into Triangular Fuzzy Numbers (TFN). The results of the study show that out of the 4 Decision Making Units (DMUs) studied without using the fuzzy approach, only the Helvetia housing complex achieved 100% efficiency. Meanwhile, the efficiency values for Tuntungan, Martubung, and Johor housing estates were 85.92%, 88.69%, and 94.87%, respectively. When the fuzzy approach was applied, the efficiency values of the Johor and Helvetia housing estates reached 100% efficiency, while the Tuntungan and Martubung housing estates had an efficiency of 78.22%, indicating inefficiency. This inefficiency is caused by excessive use of drainage inputs, indicating that these housing complexes are unable to produce environmental outputs commensurate with the inputs used. This study recommends improving the quality and quantity of environmental management and cleanliness, as well as the availability and quality of green spaces, to enhance environmental efficiency in housing. The findings of this study provide important insights into the efficiency of environmental performance measurement and highlight opportunities for improvement in environmental management within housing.

Keyword: *Environmental Performance, Data Envelopment Analysis (DEA), Fuzzy Approach, Efficiency.*

1. Introduction

Environmental performance is the measurable result of an environmental management system related to the management of environmental aspects. Environmental performance evaluation is based on environmental programs or plans, environmental targets, and environmental support facilities [1]. Housing is a collection of houses that function as a residential environment, complete with environmental facilities (infrastructure) including the provision of drinkable water, telephone and electricity cables, garbage and household waste disposal, roads, and basic physical environmental amenities to ensure that the environment functions as it should. Measuring environmental performance is important in evaluating the efficiency of an entity, especially in the context of sustainability. Housing is one of the forms of public services vulnerable to environmental pollution impacts. Based on several studies on satisfaction levels with environmental conditions, it was found that housing serves as an indicator of environmental quality, where indicators of a safe and healthy living environment include environmental infrastructure, facilities, and utilities (PSU) that are supported. Housing consists of several groups of houses as part of a settlement, both in urban and rural areas, and is equipped with infrastructure, facilities, and public utilities [2].

One of the issues related to environmental performance today has both positive and negative impacts. Among the positive impacts are decreasing greenhouse gas emissions, improved water and air quality in urban areas, and increased biodiversity. The negative impacts include issues related to forestry and waste. The amount of plastic and medical waste generated is increasing, and deforestation is also on the rise [3]. Measuring environmental performance is important in evaluating the efficiency of an entity, particularly in the context of sustainability. Housing is equipped with basic infrastructure and public utilities, making it an indicator of environmental quality. Currently, despite improvements such as reduced emissions and increased biodiversity, issues like increased plastic waste and deforestation persist. As seen in Medan today, housing development continues in some areas and is expanding year by year. However, there has been no research on the environmental performance of these housing developments to determine whether they are efficient or not. However, environmental performance data often contains uncertainty or variability, which can affect the results of the analysis. Therefore, DEA with a fuzzy approach is applied to address this uncertainty. Using this approach, uncertain data is converted into fuzzy numbers, allowing for more flexible and realistic analysis in measuring efficiency [4].

DEA (Data Envelopment Analysis) is a nonparametric method for measuring the efficiency of a decision-making unit (DMU). DEA measurement is used to evaluate the productivity of a decision-making unit (work unit) responsible for using a number of inputs to obtain a targeted output. Simply put, the measurement is expressed as the ratio between output and input, which is a unit of productivity measurement that can be expressed partially or totally, helping to identify which input factors are most influential in producing an output [5]. The DEA method with a fuzzy approach has proven effective in addressing data variability and uncertainty, which often occur in environmental performance measurement. This approach provides a solution for handling uncertainty in data by converting data into fuzzy numbers, enabling more accurate and realistic analysis. In the context of environmental performance, this is particularly beneficial for generating more precise efficiency comparisons, especially when the data used is not always certain or stable [6]. Therefore, based on the above discussion, this study aims to analyze and measure the efficiency level of environmental performance in one of the residential areas in Medan City using the DEA (Data Envelopment Analysis) method with a fuzzy approach. This study is expected to provide insights into how environmental performance measurement efficiency is assessed in a residential area.

2. Method

This study uses a quantitative approach with the Data Envelopment Analysis (DEA) method. The type of research used in this study is descriptive research, which is used to describe environmental performance efficiency based on predetermined indicators.

This study was conducted using secondary data. Secondary data is data that has been collected and published by other parties, not obtained directly by the researcher [7]. This data typically comes from books, journals, research reports, government documents, or other official sources. The secondary data used in this study was obtained from a research report conducted by Khalida Wati titled “A Study of the Analytical Hierarchy Process (AHP) Method in Measuring Environmental Performance of Villa Zeqita Medan Tuntungan Housing During the COVID-19 Pandemic.” Then, the research conducted by Hafizah Miranda Putri Harahap entitled “Robust Regression Analysis in Performance Measurement (Case Study of Martubung Housing in Medan).” Furthermore, the research report conducted by Rumata Christella Hutapea entitled “Residents’ Housing Preferences in the Western Periphery of Medan City (Case Study: Perumnas Helvetia, Medan Helvetia District)”. And finally, the research report conducted by Emir Haris entitled “Housing Preferences of the Community in Medan Johor District, Medan City”.

DMU (Decision Making Unit) is a term used for units whose efficiency is to be measured [8]. In this case, DEA (Data Envelopment Analysis) is a technique that uses linear programming to assess the relative efficiency of decision-making units by comparing one DMU with another DMU that uses the same resources to produce similar outputs [9].

In the Data Envelopment Analysis (DEA) method, linear programming is used to determine the relative efficiency of several decision making units (DMUs). The main objective is to find the most efficient DMU among other DMUs by comparing the efficiency levels of each DMU. Efficiency is measured by comparing the inputs used with the outputs produced. If the ratio of the two is equal to 1, it indicates full efficiency, and if the ratio of the two is less than 1, it indicates inefficiency [10]. Thus, efficiency is

$$Efficiency = \frac{Output}{Input}$$

The steps in this study include:

1. Determining the DMU as well as the inputs and outputs.
2. Transforming the classified input and output values into Triangular Fuzzy Numbers (TFN) or performing fuzzification. Where the input values are expressed as $\tilde{X}_i = (l_i, m_i, u_i)$ and the output values are expressed as $\tilde{Y}_r = (l_r, m_r, u_r)$. The notation lij, mij, and uij are the lower *(l)*, most possible *(m)*, and upper *(u)* values for the i-th input variable of the j-th DMU. The notation *lrj*, *mrj*, and *urj* represent the lower *(l)*, most possible *(m)*, and upper *(u)* values for the r-th output variable of the j-th DMU.
3. Perform defuzzification of the transformed performance values using the following equation:

$$Z_{ij} = \frac{(u_{ij} - l_{ij}) + (m_{ij} - l_{ij})}{3} + l_{ij} \qquad (1)$$
$$Z_{rj} = \frac{(u_{rj} - l_{rj}) + (m_{rj} - l_{rj})}{3} + l_{rj}$$

4. Calculate DEA efficiency using the following equation
   Maximum

$$maks\theta_p = \frac{\sum_{r=1}^{m} U_r Y_{rk}}{\sum_{r=1}^{m} V_i X_{ik}} \qquad (2)$$

$$s.t. \frac{\sum_{r=1}^{s} U_r Y_{rj}}{\sum_{r=1}^{m} U_r X_{ij}} \leq 1; j = 1, 2, \ldots, n \ \ldots$$

$$U_r, V_i > 0; r = 1, 2, \ldots, s; i = 1, 2, \ldots, m;$$

   Explanation:
   θ_p = relative efficiency
   Y_{rj} = r^{th} outputs from j^{th} DMU
   X_{rj} = i^{th} inputs from j^{th} DMU
   U_r = weight of r^{th} output
   V_r = weight of i^{th} input

5. Interpretation and conclusions

3. Result and Discussion

3.1. Determining DMU, Input, and Output

In this study, the DMU consists of 85 respondents. The inputs in this study are Accessibility, Drainage, and Commercial Facilities. The outputs are Green Open Space and Waste Management. The estimated values of inputs and outputs are expressed on a scale of 1–5, where 1 = Not Satisfied, 2 = Somewhat Dissatisfied, 3 = Moderately Satisfied, 4 = Satisfied, and 5 = Very Satisfied. The estimated values of inputs and outputs for the 85 DMUs are presented in Table 1.

Table 1. Data Input And Output.

DMU	Input			Output	
	Accessibility	Drainage	Facility Commersial	Environmental Management And Cleanliness	Availability And Quality Of Green Spaces
Housing (Tuntungan)	4,28	4,26	4,14	4,22	4,31
Housing (Martubung)	3,38	3,89	3,35	3,44	3,30
Housing (Johor)	2,82	2,58	3,62	3,07	3,22
Housing (Helvetia)	4,00	2,00	3,46	4,59	4,90

3.2. Transformation to TFN

Crisp input and output values are converted into fuzzy numbers in the form of TFN (Triangular Fuzzy Number) with three parameters: lower, most possible, and upper value. This process aims to capture the uncertainty in respondents' perceptions. Then, defuzzification is performed so that the data can be used in DEA efficiency calculations. The TFN formation scale can be seen in Figure 1, and the guidelines for TFN formation are based on Table 2 below:

Table 2. Guidelines for Establishing TFN

Variable	Score Range	TFN		
		Low	Most Posibble	Upper
1 = Not satisfied	0.0 – 1.5	1	1	1
2 = Somewhat dissatisfied	1.6 – 2.9	1	2	3
3 = Somewhat satisfied	3.0 – 3.9	2	3	4
4 = Satisfied	4.0 – 4.4	3	4	5

Based on the above guideline scale, the crisp input and output estimates are converted into fuzzy numbers. The results of input and output fuzzification can be seen in Table 3.

Table 3. Fuzzification Results

DMU	Input			Output	
	Accessibility	Drainage	Facility Commersial	Environmental Management And Cleanliness	Availability And Quality Of Green Spaces
Housing (Tuntungan)	(3,4,5)	(3,4,5)	(3,4,5)	(3,4,5)	(3,4,5)
Housing (Martubung)	(2,3,4)	(3,4,5)	(2,3,4)	(2,3,4)	(2,3,4)
Housing (Johor)	(1,2,3)	(1,2,3)	(2,3,4)	(2,3,4)	(2,3,4)
Housing (Helvetia)	(3,4,5)	(1,2,3)	(2,3,4)	(4,5,5)	(4,5,5)

3.3. Defuzzification

Defuzzification is performed based on Table 3. Using equation (1). The defuzzification results can be seen in Table 4. As for the calculation example, suppose for DMU 1, which is Tuntungan housing with accessibility input variables, the defuzzification results are obtained as follows.

$$Z_{11} = \frac{(u_{11} - l_{11}) + (m_{11} - l_{11})}{3} + l_{11}$$

$$Z_{11} = \frac{(5 - 3) + (4 - 3)}{3} + 3$$

$$Z_{11} = 4{,}00$$

Table 4. Defuzzification Results.

DMU	Input			Output	
	Accessibility	Drainage	Facility Commersial	Environmental Management And Cleanliness	Availability And Quality Of Green Spaces
Housing (Tuntungan)	4,00	4,00	4,00	4,00	4,00
Housing (Martubung)	3,00	3,00	3,00	3,00	3,00
Housing (Johor)	2,00	2,00	3,00	3,00	3,00
Housing (Helvetia)	4,00	2,00	3,00	4,67	4,67

3.4. DEA Efficiency Calculation

DEA efficiency calculation results using STATA and MaxDEA software. The results obtained from equation (2) can be seen in Figure 1, 2 and 3 below.

```
options: RTS(CRS) ORT(OUT) STAGE(2)
CRS-OUTPUT Oriented DEA Efficiency Results:
                                                                ref:                         ref:
                            rank                theta
dmu:                           4              .859243                  .                        .
dmu:                           3              .886929                  .                        .
dmu:                           2              .948717                  .                        .
dmu:                           1                    1                  .                        .

                            ref:                 ref:             islack:                  islack:
                                                               aksebilitas                drainase
dmu:                           .               .91939                  .                   1.8216
dmu:                           .              .749455                  .                  1.95124
dmu:                           .              .668845                  .                     1.11
dmu:                           .                    1                  .                        0

                         islack:               oslack:                 oslack:
               fasilitaskomersial  manajementlingkungankebersihan  ketersediaandankualiatasruangter
dmu:                     .376176             2.16e-07                 .195011
dmu:                     .378098             2.54e-08                 .372331
dmu:                     1.12015                    0                .0573418
dmu:                           0                    0                       0
      name:  dealog
       log:  C:\Users\User\Documents\dea.log
  log type:  text
 closed on:  18 Jun 2025, 10:32:05
```

Figure 1. DEA Analysis Results without Fuzzy Approach using STATA

DEA analysis without the fuzzy approach shows that only DMU 1 is fully efficient (score of 1), while the other three DMUs are not yet efficient, with the highest score for DMU 2 (0.9487) and the lowest for DMU 4 (0.8592). The inefficiency is primarily due to excess input in the drainage variable, with slack ranging from 1.11 to 1.95. In addition, there is a shortage of output in the environmental management and green open space variables, especially in DMU 4. DMU 1 serves as a benchmark for the other DMUs. These results form the basis for comparison with the fuzzy DEA model in assessing the effect of data uncertainty on efficiency.

```
options: RTS(CRS) ORT(OUT) STAGE(2)
CRS-OUTPUT Oriented DEA Efficiency Results:
                                                                ref:                         ref:
                            rank                theta
dmu:                           4              .782269                  .                        .
dmu:                           3              .782269                  .                        .
dmu:                           1                    1                  .                        .
dmu:                           1                    1                  .                        .

                            ref:                 ref:             islack:                  islack:
                                                               aksebilitas                drainase
dmu:                     .521512              .521513                  .                  1.04303
dmu:                     .391134              .391134                  .                  .782269
dmu:                           1                    0                  .                        0
dmu:                           0                    1                  .                        0

                         islack:               oslack:                 oslack:
               fasilitaskomersial  manajementlingkungankebersihan  ketersediaandankualiatasruangter
dmu:                    2.80e-07                    .                       0
dmu:                    2.10e-07                    .                       0
dmu:                           0                    .                       0
dmu:                           0                    .                       0
      name:  dealog
       log:  C:\Users\User\Documents\dea.log
  log type:  text
 closed on:  18 Jun 2025, 09:50:44
```

Figure 2. DEA Results with the Fuzzy Approach Using STATA

Of the four DMUs analyzed, two (DMU 1 and DMU 2) were declared efficient with a score of 1. DMU 3 and DMU 4 were not yet efficient, each only achieving 78.2% of the maximum output that should have been achieved. The inefficiency is primarily due to excess input in the drainage variable, with slack values of 1.043 (DMU 3) and 0.782 (DMU 4). Meanwhile, the slack in output is very small and not the primary cause of inefficiency.

DMU	Score	Rank	Benchmark (lambda)	Proportionate Movement					Slack Movement				
				Aksebilitas	Drainase	Fasilitas Komersial	Manajemen Lingkungan & Kebersihan	Ketersediaan dan Kualiatas Lingkungan Hijau	Aksebilitas	Drainase	Fasilitas Komersial	Manajemen Lingkungan & Kebersihan	Ketersediaan dan Kualiatas Lingkungan
Perumahan 1 (Vila tuntungan)	0,782	3	Perumahan 3 (Johor)(0,667);Perumahan (Helvet)(0,667)	0	0	0	1,113	1,113	0	-1,333	0	0	0
Perumahan 2 (martubung)	0,782	3	Perumahan 3 (Johor)(0,5);Perumahan (Helvet)(0,5)	0	0	0	0,835	0,835	0	-1	0	0	0
Perumahan 3 (Johor)	1	1	Perumahan 3 (Johor)(1)	0	0	0	0	0	0	0	0	0	0
Perumahan (Helvet)	1	1	Perumahan (Helvet)(1)	0	0	0	0	0	0	0	0	0	0

DMU	Projection					Dual Price				
	Aksebilitas	Drainase	Fasilitas Komersial	Manajemen Lingkungan & Kebersihan	Ketersediaan dan Kualiatas Lingkungan Hijau	Aksebilitas	Drainase	Fasilitas Komersial	Manajemen Lingkungan & Kebersihan	Ketersediaan dan Kualiatas Lingkungan Hijau
Perumahan 1 (Vila tuntungan)	4	2,667	4	5,113	5,113	-2,09E-01	0	-1,11E-01	2,50E-01	0
Perumahan 2 (martubung)	3	2	3	3,835	3,835	-2,78E-01	0	-1,48E-01	3,33E-01	0
Perumahan 3 (Johor)	2	2	3	3	3	-2,78E-01	0	-1,48E-01	0	3,33E-01
Perumahan (Helvet)	4	2	3	4,67	4,67	-1,79E-01	0	-9,49E-02	0	2,14E-01

Figure 3. DEA Results Using MAXDea

Two DMUs (Johor Housing and Helvet) were declared fully efficient (score = 1), while Tuntungan and Martubung Villas were not yet efficient (score = 0.782), similar to the fuzzy DEA results with STATA. Inefficiency occurred due to excess input in the drainage variable. Tuntungan had a drainage slack of 1.333 and Martubung had a drainage slack of 1. To become efficient, Tuntungan needs to increase its environmental output from 4 to 5.113 and reduce its drainage input from 4 to 2.667. Martubung needs to increase its output from 3 to 3.835 and reduce its drainage from 3 to 2. These two DMUs refer to the performance combination of Johor and Helvet as a benchmark. Dual price analysis shows that environmental management and cleanliness variables have the highest positive contribution to efficiency, while accessibility and commercial facilities variables have a negative impact.

4. Conclusion

This study aims to analyze and measure the level of environmental performance efficiency in residential areas in Medan City using the Data Envelopment Analysis (DEA) method combined with a fuzzy approach. The method used is a quantitative approach with secondary data obtained from previous research reports. Fuzzification is performed to address uncertainty in the data, where input and output values are converted into Triangular Fuzzy Numbers (TFN).

The results of the study show that out of the 4 Decision Making Units (DMUs) studied without using the fuzzy approach, only the Helvetia housing complex achieved 100% efficiency. Meanwhile, the Tuntungan, Martubung, and Johor housing complexes had efficiency values of 85.92%, 88.69%, and 94.87%, respectively. When the fuzzy approach was applied, the efficiency values of the Johor and Helvetia housing estates reached 100%, while the Tuntungan and Martubung housing estates had an efficiency of 78.22%, indicating inefficiency. This inefficiency is caused by excessive use of drainage inputs, indicating that these housing complexes are unable to produce environmental outputs commensurate with the inputs used. This study recommends improving the quality and quantity of environmental and cleanliness management, as well as the availability and quality of green spaces, to enhance environmental efficiency in housing. The findings of this study provide important insights into the efficiency of environmental performance measurement and highlight opportunities for improvement in environmental management within housing.

References

[1] Purwanto, A. T. (2003). *Pengukuran Kinerja Lingkungan*. Diakses pada 29 April 2025 https://adoc.pub/pengukuran-kinerja-lingkungan.html

[2] Wati Khalida. (2024). *Kajian Metode Analytical Heirarchy Process (AHP) pada Pengukuran Kinerja Lingkungan Perumahan Villa Zeqita Medan Tuntungan di Masa Pandemi Covid-19*. SEPREN: Journal of Mathematics Education and Applied, 5 (2): 47-56.

[3] Suryani A.S, (2020). *Dampak Pandemi Covid-19 Terhadap Lingkungan Global*. Info Singkat, 12.

[4] Putra, I. G. A., dan Arifin, Z. (2021). *Penerapan Metode Fuzzy DEA untuk Pengukuran Efisiensi Kinerja Lingkungan*. Jurnal Sistem dan Informatika, 10(3): 231-245.

[5] Fatimah Siti dan Umi Mahmudah. (2017). *Data Envelopment Analysis (DEA): Pengukuran Efisiensi Kinerja Sekolah Dasar*. Cakrawala Pendidikan: 2. 10.21831/cp.v36i2.11511

[6] Fahmi, M. A., dan Harsoyo, P. (2020). *Penerapan Metode Fuzzy DEA dalam Pengukuran Efisiensi Kinerja Lingkungan pada Sektor Industri*. Jurnal Manajemen dan Bisnis Indonesia, 17 (2): 98-110.

[7] Hardani dkk. (2020). *Metode Penelitian Kualitatif & Kuantitatf*. Yogyakarta: CV. Pustaka Ilmu Group. hlm. 401

[8] Sa'diyah Nur Halimah. (2016). *Analisis Efisiensi Menggunakan Metode Data Envelopment Analysis (DEA)*. SAINS: Jurnal Manajemen dan Bisnis, 9 (1): 101-119

[9] Tsai, PF., Molinero, CM., (2002). *A Variable Returns To Scale Data Envelopment Analysis Model For The Joint Determination Of Efficiencies With An Example Of The UK Health Service*. European Journal of Operational Research 141 21–38

[10] Giyanti Ida dan Anita Indrasari. (2018). Model Pengukuran Efisiensi Perusahaan Berbasis Fuzzy Data Envelopment Analysis. Seminar dan Konferensi Nasional IDEC Surakarta.