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Uniform Convergence of Cosine Series With Coefficient from New Class of General Monotone

Moch Aruman Imron

Department of Mathematics, University of Brawijaya, Malang, Indonesia

Abstract. General monotone sequences class (GMS) has been introduced by Tikhonov. Then Bogdan Szal extended this class to the new class called new class of general monotone. By using non-negative sequence is called β to controle the difference of sine series coefficient. By substituting β of modulus of sine series coefficient, we study uniform convergence of new class of general monotone on cosine series.

Keyword: New Class General Monotone, Cosine Series, Uniform Convergence

Abstrak. *Kelas general monotone sequences (GMS) telah diperkenalkan oleh Tikhonov. Lalu Bogdan Szal memperluas kelas ini ke kelas baru disebut new class of general monotone. Dengan menggunakan barisan non-negatif yang disebut β untuk mengatur selisih dari koefisien deret sinus. Dengan mensubstitusi β dari modulus koefisien deret sinus, kami pelajari konvergen seragam dari new class of general monotone pada deret kosinus.*

Kata Kunci: *New Class General Monotone, Deret Kosinus, Konvergen Seragam*

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1. Introduction

Chaundy and Jollife [6] have been discussed about the classic theorem, the necessary and sufficient condition of uniform convergence of

$$\sum_{n=1}^{\infty} a_n \sin nx \tag{1}$$

is $\lim_{n\to\infty} na_n = 0$, if $a = \{a_i\}$ is decreasing monotone and tending to zero. Tikhonov extended decreasing monotone (1) to General Monotone (GM) by Tikhonov [5].

Definition 1.1. *Let $a = \{a_i\}$ be a complex sequence and let $\beta = \{\beta_i\}$ be a non-negative real sequence, i.e. $\beta_i \geq 0$ for each $i \in \mathbb{N}$. The ordered pair (a, β) is in β general monotone, if there exists positive constant C and the following inequality stands for every $n \in \mathbb{N}$*

*Corresponding author at: University of Brawijaya, Malang, Indonesia

E-mail address: maimr@ub.ac.id

$$\sum_{k=n}^{2n-1} |a_k - a_{k+1}| \leq C\beta_n$$

Bogdan Szal [1] extended class of general monotone called class (β, r) general monotone.

Definition 1.2. *If* $a = \{a_i\}$ *complex sequence numbers and* $\beta = \{\beta_i\}, \beta_i \geq 0$ *for each* $i \in \mathbb{N}$. *An ordered pair* (a, β) *is element in the class of ordered pair* (β, r) *of general monotone* $(GM(\beta, r))$, *if we can find some positive constant* C *and the inequality*

$$\sum_{k=n}^{2n-1} |a_k - a_{k+r}| \leq C\beta_n$$

holds for each $n \in \mathbb{N}$ *and* $r \in \mathbb{N}$.

For

$$\beta^* = \beta^*(r) = \sum_{k=n}^{n+r-1} |a_k| + \sum_{k=\left[\frac{n}{c}\right]}^{[cn]} \frac{|a_k|}{k},$$

and some $c > 1$ and it can be obtained some result at follows

Theorem 1.3. *Let* $a = \{a_i\}, a_i \geq 0$ *for each* $i \in \mathbb{N}$ *and* $(a, \beta^*) \in GM(\beta^*, r)$, *where* r *is natural numbers. If* $r \geq 1$ *and the sine series in point* (1) *is convergent uniformly to continuous function, then* $\lim_{n\to\infty} na_n = 0$.

Theorem 1.4. *Let* $a = \{a_i\}, a_i \geq 0$ *for each* $i \in \mathbb{N}$ *and* $(a, \beta^*) \in GM(\beta^*, r)$, $r \in \mathbb{N}$. *If* $r = 2$ *and* $\lim_{n\to\infty} n|a_n| = 0$, *then the sine series in point* (1) *converges uniformly.*

Throught this paper, by using a new class of the general monotone [1] and let $\beta_n = a_n \geq 0$, we discuss uniform convergence of this new class on cosine series like as [2], [3], [4].

2. Materials and Methods

This research is worked by study of literature and supporting paper from journals to get an understanding, then get results related to the research that published in the scientific journal. The results of this research are communicated in a journal. In summary the method of the research is discussing uniform convergence with coefficient belonging to the new class of the general monotone.

3. Results and Discussion

In this section, we will discuss some modification of new class general monotone by taking a non-negative real sequence for controlling the sum of difference sine series coefficient. Let the series

$$f(x) = \sum_{k=1}^{\infty} a_k \cos kx \tag{3.1}$$

where $a = \{a_k\}$ is a real sequence that tends to zero, i.e., $a_k \to 0$ as $k \to \infty$. We define sums of series *f(x)* where the point of the series converges.

Definition 3.1. *A non-negative real sequence $a = \{a_i\}$ is called element of class of $GM(\mathbb{R}^+, 2)$ if we can find positive constant C, i.e. $C > 0$ and the relation*

$$\sum_{k=n}^{2n-1} |a_k - a_{k+2}| \leq C\boldsymbol{a_n}$$

holds for each $n \in \mathbb{N}$.

Lemma 3.2. *If $\boldsymbol{a} \in GM(\mathbb{R}^+, 2)$* and *$\{na_n\}$ converges to 0, then*

$$\lim_{n\to\infty} n \sum_{v=n}^{\infty} |a_v - a_{v+2}| = 0$$

Proof. Let $d_n = \sup_{v \geq n}(va_v)$, then d_n decreasing monotone and $\lim_{n\to\infty} d_n = \lim_{n\to\infty} ma_m = 0$. For $a \in M(\mathbb{R}^+, 2)$, we have

$$n \sum_{v=n}^{\infty} |a_v - a_{v+2}| = n \sum_{s=0}^{\infty} \sum_{v=2^s n}^{2^{s+1}n-1} |a_v - a_{v+2}|$$

$$\leq \sum_{s=0}^{\infty} 2^s n \frac{a_{2^s n}}{2^s} \leq \sum_{s=0}^{\infty} \frac{na_n}{2^s} = 2C'na_n.$$

Thus

$$\lim_{n\to\infty} n \sum_{v=n}^{\infty} |a_v - a_{v+2}| = 0. \qquad \blacksquare$$

Theorem 3.3. *Let $a \in GM(\mathbb{R}^+, 2)$ and $\lim_{n\to\infty} na_n = 0$, if $\sum_{k=1}^{\infty} a_k$ converges then cosine series* (1) *converges uniformly on closed interval $[0, \pi]$.*

Proof. Since $\sum_{k=1}^{\infty} a_k$ converges and $\{a_k\}$ is a null and non negative sequence, then

$$\lim_{n\to 0} \sum_{k=n}^{\infty} a_k = 0. \tag{3.2}$$

Let us calculate $f(x) - S_m(f, x)$, where

$$S_m(g, x) = \sum_{j=1}^{m} a_j \cos jx$$

we get

$$f(x) - S_{m-1}(f,x) = \sum_{k=m}^{\infty} a_j \quad \cos jx \tag{3.3}$$

$$= \sum_{k=m}^{\infty} \Delta a_k D_k(x) - a_m D_{m-1}(x) \ = A + B$$

where $D_m(x) = \sum_{j=1}^{m} \cos jx$, $A = \sum_{k=m}^{\infty} \Delta a_k D_k(x)$ and $B = -a_m D_{m-1}(x)$.

Let x in interval $(0,\pi]$, so we can find natural number M satisfying $x \in \left(\frac{\pi}{M}, \frac{\pi}{M-1}\right]$. Because of

$$\left|\sum_{v=1}^{m} \cos vx\right| = \left|\frac{\sin\left(m+\frac{1}{2}\right)x}{2\sin\frac{1}{2}x}\right| \leq \frac{1}{2\sin\frac{1}{2}x} \leq \frac{\pi}{2x}, \tag{3.4}$$

we have

$$|B| = |a_m D_{m-1}(x)| \leq \frac{\pi}{2x}|a_m| \leq \frac{M}{2}|a_m|.$$

By (3.4) and a_i nonnegative for each $i \in \mathbb{N}$, we obtain

$$|A| = \left|\sum_{k=m}^{\infty} \Delta a_k D_k(x)\right| \leq \frac{\pi}{2x}\left|\sum_{s=m}^{\infty} \Delta a_s\right| \leq \frac{M}{2}\left|\sum_{s=m}^{\infty} \Delta a_s + \sum_{s=m+1}^{\infty} \Delta a_s\right| \leq \frac{M}{2}\sum_{s=m}^{\infty}|a_s - a_{s+2}|$$

By proof of Lemma 3.2 we have

$$|A| \leq MCa_m.$$

Thus we get

$$A + B \leq \frac{M}{2}(a_m + 2Ca_m) \tag{3.5}$$

For $x = 0$ we have

$$f(x) = \sum_{k=1}^{\infty} a_k \cos 0 = \sum_{k=1}^{\infty} a_k < \infty,$$

finally by (3.2) and (3.5), we get

$$\lim_{n\to\infty} f(x) - S_m(f,x) = 0.$$

Then the series in point (3.1) converges uniformly on closed interval $[0,\pi]$. The proof is complete. ∎

4. Conclusion

In this paper we have concluded that, necessary and sufficient condition of uniform convergence of cosine series with coefficient in new class of general monotone $GM(\mathbb{R}^+,2)$ is $\lim_{n\to\infty} nb_n = 0$.

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