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Mathematical Philosophy

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Abstract. Mathematics and philosophy are two words with different meanings and the same thing. With various historical evidence, mathematics as the basis of science is not part of or born from philosophy. In the same position in knowledge, mathematics confirm the answers to intimate problem in philosophy. Often there is confusion in philosophy because of conflicting concepts with one another. Mathematics without philosophy does not move swiftly, because without the meanings that are sometimes driven by philosophy. Logically, truth is not well developed in evidence except when mathematics and philosophy get long. It is to provide an understanding of the need for a foundation of truth thought, which generally reveals in the comprehension of mathematics, namely in meta-mathematics and philosophy.

Keyword: Arithmatic, Algebra, Geometry, Meta-mathematics, Number, Paradox, Trigonometry.

Abstrak. *Matematika dan filsafat adalah dua kata yang berbeda dan dengan makna yang sama. Dengan berbagai bukti sejarah, matematika sebagai dasar, dari bidang keilmuan yang lain, tidaklah bagian atau lahir dari filsafat. Pada posisi yang sama dalam pengetahuan, matematika mengukuhkan jawaban terhadap persoalan-persalan yang mendalam di dalam filsafat. Sering terjadi dalam filsafat pertentangan sebagai sebab konsep yang tidak saling sejalan, atau kacau. Matematika pula tanpa filsafat tidak bergerak dengan tangkas, karena tanpa makna-makna yang kadangkala didorong oleh filsafat. Secara nalar, kebenaran tidak berkembang dengan baik dalam bukti kecuali ketika matematika dan filsafat dapat bersama. Tulisan ini untuk memberikan pemahaman kepada perlunya landasan pemikiran atau kebenaran, yang umumnya terungkap dalam pemahaman terhadap matematika. Pemahaman itu berasal dari konsep bertanya yang menjadi budaya dalam berbahasa, yaitu adi-matematika dan filsafat.*

Kata Kunci: *Aritmatika, Aljabar, Geometri, Adi-matematika, Bilangan, Paradoks, Trigonometri.*

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1. Introduction

Mathematics, something that is difficult to interpret when it is just a word that expresses the name of knowledge only [1]. Mathematics conceptually requires reasoning that follows the language in which it expresses [2]. Mathematical language formulation is not only a pointer to

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the basis of science but also has its essence [3]. Essence shapes mathematics itself, and its extension to various branches of science [4]. Naturally, mathematics has different names in different language [5, 6]. It is following the designation and understanding and the interests of the humans who use them. In principle, different names have the same essence [7].

Mathematics justifies something in abstraction following reason. Apart from mathematics, philosophy is a knowledge that seeks ultimate truth, although always incomplete [8]. The combination of these two words produces an expression about the philosophy of mathematics [9,10], which encourages an understanding of mathematics and philosophy [11]. The paradox is an example of the problem of interaction between mathematics and philosophy, which makes it possible to reveal the mathematical philosophy [12]. That way, it’s possible to understand the questions [13]. Philosophy may be able to answer many problems in-depth, but also many contradictions [14], mathematics can answer thoroughly with proofs [15]. The description of this starts from an understanding of mathematics and philosophy, through history and thinking. Meta-mathematics tries to answer the need for the methodology of mathematics. In the end, it is to complete the questions through discussion to find differences or determine the similarities between mathematics and philosophy.

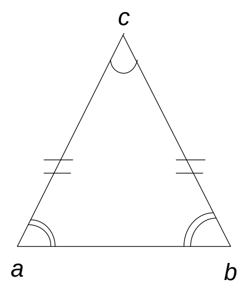

Figure 1 An isosceles triangle.

2. A Review: History as a Proof

Call it as mathematics for naming what it describes as follows. The term that comes from Greek, namely μαθημα (read *máthēma*) has a meaning as a science that deals with the structure, arrangement, and relationships that involve the calculation, measurement, and formulation of forms [2]. Mathematics based on the oldest traces presents abstract geometry as preliminary evidence, namely following the common sense practiced by the Babylonians and Egyptians over the centuries [16]. The work systemized by one of the seven wise men of Greece, named Thales, from Miletus (located on the western coast of present-day Turkey) [17, 18]. He had turned the practical instructions of this practice measurement into step-by-step mathematical proof as found in current measurement science [19]. Thales proved six basis propositions of geometry. The proposition that the two base angles of an isosceles triangle are equal, see Figure 1. On the other hand, Thales was looking for a single element upon which to change or shape the universe [20]. As one point becomes a line representing another point, this essence expresses the number adjacent to the geometry [21]. For example, the distance between two points on a plane becomes a measure in units. Logically, the development of human thought is formed through patterns of forms that exists, recognizes them, and expresses them in other ways, as geometry represents numbers [22]. Many computation patterns, following numbers, in everyday life exist with humans. Each second builds up the minute, the minute builds up the hour, where one-hour passes and the next hours tell up to one day [23]. Seven days add up in one week, or between 28 and 31 days to form a month. Month after month runs until the limit of the number of days is one year [24]. Onwards, terms that describe numbers indirectly, form decades, centuries, and so on [25].

Thales also questioned the origin, nature, and structure of the universe [26]. The unit of knowledge is known as cosmology, which was also originally referred to as natural philosophy [27, 28]. Philosophy as a form of human effort to understand deeply about something, namely a term that comes from Greek, namely φιλοσοφια (read *philosophia*) which means love of wisdom, or the study of general problems and fundamental to existence, knowledge, values, reason, thought, and language [29]. Based on the concept of existence, as a scientist, Thales had understood that the moon shines because it reflects light from the sun [30]. Indirectly, it reveals that mathematics and philosophy exist together.

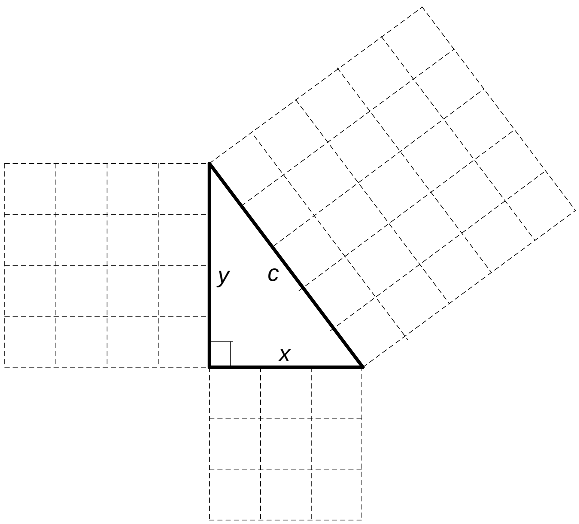

Figure 2 A right triangle.

The second wise man, named Pythagoras [31], has engraved his name in the postulates of a right triangle [32], which reveals that "*the sum of the square of the two sides of a right triangle is equal to the square area of the hypotenuse*." Figure 2 is a proof without words [33], which reveals a calculation like the following formula

$$c^2 = x^2 + y^2. \tag{1}$$

The formula in Eq. (1) had known by every school student as Pythagoras' argument, where mathematical proof step-by-step has published in the book ***Elements*** compiled by Euclid [34]. Thus, the concept of number comes with geometry. In other words, the followers of Pythagoras stated that numbers are the essence and basis of the properties of things, including all forms, such as triangles, squares, rectangles, hexagons, and others [35]. This philosophy that prioritizes numbers condensed into a proposition that reads "*Number rules the universe*. [36]"

The close relationship between mathematics and philosophy is reflected geometry in particular and while number theory exists naturally [37]. Plato states that God always works by geometrical methods [38], and then C. G. J. Jacobi completes the statement with a phrase: "*God ever arithmetizes*. [39]" However, at first, humans will know something more easily through forms (as in geometry), then reveal its properties (as in arithmetic) [40]. In others words, that human respond to all that the human senses recognize on a case-by-case basis, but then abstract them in thought. It is in line with the opinion of the astronomer and physicist James H Jeans, who stated that "*the Great Architect of the Universe now begins to appear as a pure mathematician.* [41]"

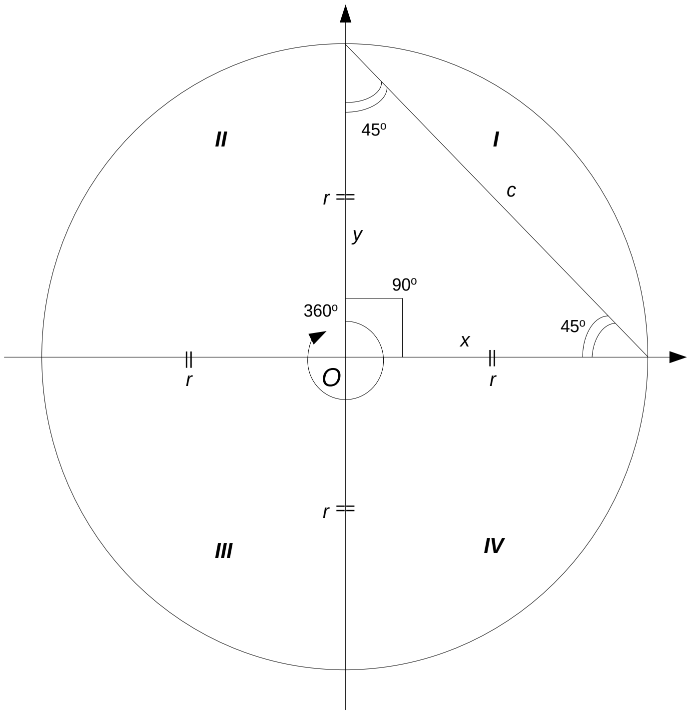

Figure 3 A circle, a right triangle, quadrants, and coordinate system.

Either the relationship or interaction between mathematics and philosophy, as expressed in the formulation of geometry and arithmetic by Thales and Pythagoras, reveal differences and similarities [42]. The term arithmetic from Greek, consisting of two words αριθμος (read *arithmos*) means number and τέχνη (read *tické* or *techne*) means the art of science. Arithmetic is the basic part of number theory, or the branch of mathematics which consists of the study of numbers, the properties of its traditional operations [2].

The interaction between mathematics and philosophy reveals the equivalent of concepts, phenomena, and paradigms in each of this knowledge [43]. Mathematicians study in the abstraction of the notions of infinity, possibility, and numbers, while the philosophers contemplate immortality, change, and quantity [44]. In the learning concept, there is a relationship between immortality and infinity [45]. In the search implication, there is always a relationship between change and possibility [46]. Or, in the research application, there is a relationship between quantity and number [47]. The concept of time, such as one hour = 60 minutes or one hour = 360 seconds, for example, inspires that a circle as a geometric shape consists of 360 degrees (written as $360°$), as indicated by timepieces in various places, or that all geometrical shapes related with size $360°$ [48]. With the presence of the number 360 reveals a field of mathematics, which philosophically, in logic it is the reason for dividing the circle into $360°$. It causes a field of mathematical science based on numbers and geometry, namely trigonometry [49]. The Sumerian astronomer, named Hipparchus, had compiled trigonometric tables as a means of explaining different numerical forms [50]. Trigonometry generally involves the y and x coordinate systems, as shown in Figure 3. A circle consists of $360°$, radius r, and a coordinate system can divide it into four quadrants. At the origin, O, the abscissa, between x-axis and y-axis is perpendicular, and there are quadrants I, II, III, and IV, each at point O has an angle of 90° [51].

Trigonometry is one of the mathematical fields that study the relationship between the angle and the abscissa of the x and y axes, respectively or not. The concept of sine defines the relationship of the angle to the y-axis and the hypotenuse, i.e.

$$\sin\theta = y/r. \qquad (2)$$

Generally, the hypotenuse is as the side of the opposition with the angle 90° of the right triangle. Specifically, the hypotenuse is the radius of the circle forming the angle. Thus, a function, Eq. (2) that reveals a ratio between the side opposite the angle and the hypotenuse [52]. That is one that compares the y-axis and the side of right triangle r. The cosine concept expresses the relationship of the angle to the x-axis and the hypotenuse or radius of the circle forming the angle, i.e.

$$\mathrm{con}\,\theta = x/r. \qquad (3)$$

Eq. (3) is a function expressing the ratio of the adjacent leg and to the hypotenuse. That is a comparison of the x-axis and the side of the right triangle [53]. Meanwhile, the relationship between the angle and the y-axis and the x-axis represents $\tan\theta = y/x$ which also states $\tan\theta = (\sin\theta)/(\mathrm{con}\,\theta)$. It is a function that defines the ratio between the opposite leg and the adjacent leg. So, even though Hipparchus does not record as a philosopher, trigonometric regularity comes from the ontology [54]. A methodology in philosophy that systemizes the scattered concepts in geometry and numbers, where ontology reveals the existence of mathematical objects such as numbers, sets, functions, and others [55]. This equivalence states that mathematics is not part of or derived from philosophy, and that philosophy was originally part of mathematics based on proofs in Table 1. It has revealed aspects of the similarities and differences between mathematics and philosophy. According to Plato, geometry follows pure reason, and philosophy is said to employ reason solely where the continuation of each does not perform experiments and does not require laboratory equipment [56].

What mathematics and philosophy have in common is that they operate at a high rate of increment [57]. They both discuss very general ideas and usually go beyond one concrete level after another. There is no question about the existence of an object, other than the abstraction of that object [58]. For example, in mystical, the Pythagoreans believed that the number 1 (one) represented reason, number 2 (two) represented man, number 3 (three) intended to designate woman, the number 4 (four) showed justice as a result of the product of two equal numbers. So, the number 5 (five) considered to reflect marriage, namely the addition of the number two and number three as a combination of man and woman. So, mathematics does not discuss, for example, about number 2 (two) as man representation or others but the concept of numbers in general, as philosophy does not question man or woman but humans in general [35]. However, even though mathematics and philosophy both involve rationality and use rational methods. Philosophy freely involves any rational [59, 60]. The philosopher can contemplate anything as long as it is part of the human experience [61]. Meanwhile, mathematics relies on the logical method of deduction [62]. Mathematics focuses on matters relating to both numbers and spaces, relationships, patterns, shapes, and assemblies or structures [63]. Philosophy proves by involving dialogue with reasons that can make sense. Thus, philosophy tends to produce the meaning of a language with various interpretations and produce different things [64]. The development of a language, in terms of vocabulary, tends to increase. Meanwhile, mathematics proves something by studying from concept to theory, which is always an abstraction, with symbols that define in one meaning and same interpretation [65]. All that has been expressed by a philosopher named Alfred Cyril Ewing. It is of [66]: "*Firstly, it has not proved possible to fix the meaning of terms in the same unambiguous way in philosophy as in mathematics, so that their meaning is liable imperceptibly to change in the course of an argument and it is very difficult to be sure that different philosophers are using the same word in the same sense. Secondly, it is only the sphere of mathematics that we find simple concepts forming the basis of a vast number of complex and yet rigorously certain inferences. Thirdly, pure mathematics is hypothetical, i.e. it cannot tell us what is the case in the actual world, for example, how many things there are in a given place, but only what will be the case if so-and-so is true, e.g. that there will be 12 chairs in a room if there are* 5+7 *chairs. But philosophy aims at being* *categorical, i.e. telling us what really is the case; it is therefore not adequate in philosophy, as it often is in mathematics, to make deductions merely from postulates or definitions.*"

When the principles of philosophy scatter according to the thoughts of the philosophers and then they contradict one another [67, 68]. Mathematics integrates with its reflection and requires philosophy to be able to understand the operation of though in mathematics, however, in this case philosophy is a series of thoughts that carefully consider a matter in thought. Based on that reason, mathematical philosophy becomes necessary? The answer lies in the understanding given by Galileo Galilei as follows [69]: "*Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics.*"

Table 1 The Inventors of Mathematics and Philosophy

Personal	**Description**
Thales (640-546 BC)	The father of Philosophy & Deductive reasoning (Geometry)
Pythagoras (572-497 BC)	Numbers and Philosophy
Zeno (±490-430 BC)	Dichotomy & Achilles Problems (Philosophy & Mathematics)
Plato (427-347 BC)	Geometry and Philosophy

3. Meta-mathematics

Despite having difficulty in understanding the relationship between mathematics and philosophy, let alone the mathematical philosophy, or called it as a philosophy of mathematics [70]. "*A philosophy of mathematics might be described a viewpoint from which the various bits and pieces of mathematics can be organized and unified by some basic principles.*" Therefore, the mathematical philosophy is [71] "*the study of the concepts of and justification for the principles used in mathematics*."

Mathematics, besides being formalized, also formalizes mathematical proof for all matters related to understanding abstraction [72]. But for expressing mathematics self requires understanding and relationships. It is independent of the hypotheses and assumptions that often share with methodology, namely matters relating to the nature of scientific explanation, the logic of discovery, probability theory, and measurement theory. Mathematics is also not related to ontology, which discusses the concepts of substance, process, time, space, causality, the relationship between reason and matter, and the status of theoretical entities [73]. Meanwhile, there is a separate methodology for mathematics known as meta-mathematics, which is outside mathematics [74]. A formalized theory of proof includes a system of symbolic logic. Of course, Hilbert's theory of proof states the following [75]. "*The formalization of mathematical proof by means of a logistic system makes possible an objective theory of proofs and provability, in* *which proofs are treated as concrete manipulations of formulas (and no use is made of meanings of formulas)*." As a paradigms [76]: "*meta-mathematics is a branch of mathematical logic which studies formal theories and solves problems pertaining to such theories*."

The symbolic logic system complements mathematics as a complete device that forms the basis of science and other knowledge. Completeness is through the power of human thinking in the form of statements based on truth, and to express that foundation of truth sometimes through sentences that try to interpret the meaning deeply, maybe in dialogue, in questions, and so on.

4. Dialog in Questions

The abstraction that follows thought ultimately presents a paradox. Some interactions of mathematics and philosophy give rise to premises that are not systematic or chaotic. It considered truth as a basis for conclusions then has no rationale that is when the statement with its truth contradicts something else. The dichotomous oddity is an early example in this discussion. According to Zeno motion is not possible. When a moving object reaches a certain distance, it must travel $1/2$ from that distance, and before traveling half that distance, it must also cross $1/2$ the previous distance again. And so on every time there is $1/2$ of distance that must be passed continuously. It means that there is a space divided into a dichotomy of an infinite number (…), which makes it impossible to travel within a certain period. Based on that, moving from one point to another is not possible. However, the notion of a limit to infinite series provides no answer. When many numbers generally point to one point, it is called the convergence process, which will be a limit [77], which is the sum of the series. The limit of infinite series s_n is

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \big(a(1-r^n)\big)/(1-r) = a/(1-r). \quad (4)$$

Therefore, dichotomy peculiarities are not the longer paradoxical today. The following solution reveals: If the object moves 1 (one) meter from one point to another, the infinite series becomes $1/2 + 1/4 + 1/8 + \cdots + 1/(2^n) + \cdots = k$, where the first term a is $1/2$ and differentiator r is $1/2$. Based on Eq. (4), $(1/2)/(1-1/2) = (1/2)/(1/2) = 1$, and therefore $k = 1$.

Achilles' peculiarity is a paradox that is almost similar to a dichotomous oddity. This paradox reveals Achilles's sprinter is unlikely to catch up with a slow tortoise if it has overtaken a certain distance. Zeno argues that by the time Achilles reached the turtle's first point of departure, the beast had advanced a certain distance. When Achilles chased the tortoise, the tortoise had advanced again and again. It resulted in the tortoise always in front of Achilles. This paradox concerns not only distance but also time, also known as velocity. In this case, supposing the Achilles runner can cover 1 (one) km in 1 (one) minute and the turtle is only half its speed but has already moved 1 (one) km ahead, then the Achilles race with the tortoise can also be expressed as a time series as follows: $1 + 1/2 + 1/4 + 1/8 + \cdots + 1/(2^{n-1}) + \cdots = h$, where $a = 1$ and $r = 1/2$, and based on Eq. (4) $h = 2$.

Humans recognize forms easily where thought interprets them through the five senses. By limiting it to space and time, human language provides an understanding of the meaning of "oneness", "plurality", "infinity", "nothingness", "limitations", 'emptiness", and others which provide reasons for the questions "how much", "how many", "how wide", "how long", "how small" [78, 79]. This collection of questions is the application of human interests. In philosophy is the original thought, wherein language, "emptiness" is representative of the number 0 (zero), "oneness" is equivalent to the number 1 (one), but expression which is not directly similar to any number becomes a process like "movement" or "shift" changes to "addition" or the symbol + (addition); "backward" typically gives the meaning of the less/reduction symbol or -; "change" has different interpretations as it occurs in "division" or "multiplication". Although this though record from Greece, almost all human cultures give the same treatment.

Numbers are the key to human interpretations of what clings to human thought. However, it does not completely answer, resulting in problems, which human carelessly give reasons. Therefore, reasoning that is not philosophically ultimate, for that reason, humans often ask questions in their activities with the hint of the word "how". In philosophy, the question involves the word "how" is the most superficial reason. In the interpretation of numbers, whether measured or not, it only involves technology, which absorbs into the life of the human person as a skill. And when that person is no longer and the answer to the question "how" is lost [80]. The number means zero or "nothingness" and also nothing. In many languages, the question "how" has the same interpretation as "how …" or it is equal to "far … far away" [81]. It expresses the emphasis that represents the distance where a number can admit it, such as 1, 2, 3, … . Therefore, the philosophical level of the question using the word "how" is low, in mathematics, the questions involving the word "how" only provide a simple solution in the repetitive pattern of problems and answers in many textbooks for a teaching.

Numbers describe form to matter, such as in mathematics, the number describes further through the geometric shape trigonometry as well as an ultimate understanding will encapsulate an object. In philosophy, the question uses the word "what" comes from reason to get a continuous description of an objects. The description ensures that no explanation does not implement, even though the entity does not always reveal the meaning. The number sequence explains the peculiarity of the dichotomy and causes it to no longer be a paradox. It is as the problem the Achilles' peculiarity. In contrast, through numbers abstracting into symbols, a description in mathematics is revealed through definitions that sometimes contain axioms. Nearly all axioms, even with the emphasis on logic, contain interpretations of numbers in depth. Let us just say, $x + y = y + x$ is the abstraction of all arbitrary numbers. It is a description of the commutative law that exist in mathematics. The examples in numbers describe all the possibilities. In philosophy, the question "what" to all human experiences that follow knowledge and culture. Mathematics is not only culture but also as knowledge, which gives maturity to humans through explanations, which are sometimes in graphs that interpret numbers [82, 83]. Philosophically, when the picture is a thousand words, the numbers are often misinterpreted into the graphs.

Numbers provide ultimate reasoning when there is an abstraction into symbols. The evidence is better. As is evident, Eq. (2) against dichotomous oddity or Eq. (3) against Achilles's peculiarity. In which mathematics places itself as a reason that integrates understanding divided by philosophy. Indeed, in philosophy, using the question word "why" is to look for ultimate reasons, not just an answer or an explanation, but the arrival of the reason to the goal does not necessarily reach the truth. However, many of the results of that successive thinking are concepts that prioritize perception. In contrast, in mathematics, when the abstraction has broken down into the definitions as the initial understanding. It is reason provides a target of evidence for the "why" question. Therefore, it is a mandatory suggestion that research questions come to the point of expressing reason with "why" questions [84, 85]. The reason with the math object N as a set is that there are other math objects $\circ$ through the number description explaining

$$c = a \circ b \tag{5}$$

for all a, b, c in N. The same numbers will explain the expression

$$a \circ (b \circ c) = (a \circ b) \circ c \tag{6}$$

Likewise, the law distributive

$$a(b \circ c) = ab \circ ac, \tag{7}$$

the law identity, i.e.

$$a \circ i = a \tag{8}$$

where i is the identity, and logically there a^{-1} causes

$$a \circ a^{-1} = i \tag{9}$$

and a^{-1} in N is invers. However, for both philosophical and mathematical reasons, it is better if a statement can represent the whole question by involving all the question words [86].

In principle, when change occurs, mathematics also expresses the absence of the change. In philosophy, it expresses permanence to eternity. It is like the following arrow paradox. Arrows relate specifically to space, time, and distance. Each time an arrow released from the bow. At that point, the arrow will occupy a space exactly its size. At that moment, the arrow did not move. Call it when it is "now". So as of "now", the arrow doesn't move. So at the next moment, on the same basis, the arrow does not move. Finally, the arrow that was released from the bow was unable to move. It is the conceptual equivalent, the closure of the Eq. (5). Numerical interpretations do through mathematical objects such as the set N and the function $\circ$, namely otherwise known as binary operations. If a in N and b in N, then c in N for $c = a \circ b$. An arrow does not move at present, another arrow is also the same, so another arrow is in the same position or is in the same space. These laws of equality have existed with numbers as explanations since humans recognized numbers and conceptually abstraction through time-to-time expression until now. Algebra is the name of a collection of these concepts specifically in mathematics to pay tribute to an Islamic mathematician who carried out a systematic arrangement of these concepts, which is another field that systematic is called algorithms [87].

The answer to reasoning the question word "why" is to express things ultimately. The evidence in the statements supports one another. When the concept of algebra provides group theory with the condition that for every a, b in N there is c in N for $c = a \circ b$, that for every a, b, c in N applies $a \circ (b \circ c) = (a \circ b) \circ c$, that for every a in N there is i in N to $a \circ i = a = i \circ a$, and that for every a in N applies $a \circ a^{-1} = i = a^{-1} \circ a$ where a^{-1} in N. It is a general concept, but it applies specifically [88]. Philosophically, humans start counting from one, one, …, one, and so on, until finally, other numbers are appearing in the form of abstraction from the recognition of the all senses. All are just the two digits 1s and 0s, which are now recognized by binary numbers. This number concept interprets many human activities, ranging from life and death, and now all jobs follow the numbers based on the numbers 1 and 0. Through a combination of 1 and 9, humans can command machines (inanimate objects) to move as human move. Meaning in philosophy, something immovable can move through numbers, for example, to express the existence of an object involving 0001 in the four digits of binary number or 01h (h = hexadecimal). Thus when in algebra, there is a group based on the axiom above on Eq. (5), Eq. (6), Eq. (8) and Eq. (9). With philosophical reasoning, algorithms are functions that operate all binary arrangements in combinations in groups so that an object is capable of moving. Now recognizes that object as a computer, and there is a language to govern it. It call as machine language, just as mathematics sometimes calls the language of science and technology, but can philosophy be the language of knowledge? Thus, in philosophy, there is a truth that breaks down according to concepts and perceptions. However, these concepts are taken over naturally utilizing numbers or mathematics, and interpret them into something like meaning [89].

Mathematics, as the basis of science and knowledge, therefore, is also increasingly detaching itself from the attachment to science itself. Through meta-mathematics, which applies strict expectations, mathematics grows independently from the beginning until now through objects of study that are increasingly broad and deep in terms now through objects of study that are increasingly broad and deep in terms of human thought [90]. The interpretation of thought by numbers provides reasoning with the laboratory, which it is also exists in mathematics that drives the operation of the laboratory itself. Mathematics is a system of knowledge that stands alone. The mathematical philosophy explains the use of meta-mathematics, which reveals that mathematics has its methodology, which is not the same as any other science-based on mathematics.

5. Conclusion

Different terms, such as expressing mathematics, exist anywhere in human life, as long as thought recognizes an object which has a quantifiable interpretation. Geometry, Arithmetic, Trigonometry, and Algebra are interpretations that accumulate in mathematics. With its completeness, meta-mathematics, the accumulation more deeply integrates and propagates to all sides of human thought, which require abstraction to facilitate understanding that is the mathematical philosophy in which mathematics does not tie to it.

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