

Monte Carlo Simulation Approach to Determine the Optimal Solution of Probabilistic Supply Cost

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Abstract. Monte Carlo simulation is a probabilistic simulation where the solution of problem is given based on random process. The random process involves a probability distribution from data variable collected based on historical data. The used model is probabilistic Economic Order Quantity Model (EOQ). This model then assumed use Monte Carlo simulation, so that obtained the total of optimal supply cost in the future. Based on data processing, the result of probabilistic EOQ is \$486128,19. After simulation using Monte Carlo simulation where the demand data follows normal distribution and it is obtained the total of supply cost is \$46116,05 in 23 months later. Whereas the demand data uses Weibull distribution is obtained the total of supply stock is \$482301,76. So that, Monte Carlo simulation can calculate the total of optimal supply in the future based on historical demand data.

Keyword: EOQ Model, Monte Carlo simulation

Abstrak. Simulasi Monte Carlo merupakan suatu simulasi probabilistic dimana solusi dari permasalahannya diberikan berdasarkan proses acak. Proses acak melibatkan distribusi probabilitas dari variable data yang diperoleh dari data yang lalu. Model yang digunakan adalah model probabilistik Economic Order Quantity (EOQ). Model ini kemudian diasumsikan menggunakan simulasi Monte Carlo, sehingga diperoleh total dari biaya pemasokan yang optimal pada masa yang akan datang. Berdasarkan pemrosesan data, hasil dari probabilistik EOQ adalah \$486128,19. Setelah menggunakan simulasi Monte Carlo dimana data permintaan mengikuti distribusi normal dan diperoleh total biaya pemasokan adalah \$46116,05 pada 23 bulan kemudian. Ketika data permintaan menggunakan distribusi Weibull, diperoleh total stok pemasokan adalah \$482301,76. Sehingga, simulasi Monte Carlo dapat menghitung total pemasokan optimal pada masa yang akan datang berdasarkan data permintaan yang lalu.

Kata Kunci: Model EOQ, Simulasi Monte Carlo

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1. Introduction

Simulation is a numerical technique to do an experiment in computer involving a certain mathematical and logical model displaying business characteristic and economic system in a

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long time period [1]. Vita [2] simulation is a tool to analyze the inventory system where a demand is a random variable reflecting an uncertain demand. A survey conducted in 1978 by The Institute Management Science (TIMS) and The Operations Research Society of America (ORSA) in America informs that a simulation on the third rank after analysis of economics and statistics.

This information states that a simulation is a tool or method that can be used to solve the problem and give a solution. The reliability of simulation can face the complex problem, measure a performance from a various data and give the alternative solution quickly using computer program. One of simulation model is Monte Carlo simulation. Djati [3] stating that a Monte Carlo simulation will give an indication of several inventory that must be at store and when the order conducted.

2. Research Methodology

Monte Carlo simulation is a pattern using random number, namely random variable $U(0,1)$ used to solve stochastic and deterministic problem, the time does not play the real role. Generally the Monte Carlo simulation is more static rather than dynamic. Monte Carlo simulation involves the random number. Render, et al., [4] there are 5 steps to perform a simulation, namely:

1. Define a known probability distribution based on historical data.
2. Build a cumulative probability distribution for each variable.
3. Perform a simulation to generate random number.
4. The result from analysis or output is used as an input and evaluation.
5. Do this simulation repeatedly.

Random number is an arbitrary number and have a criteria that must be fulfilled, namely:

1. Random number has uniform distribution.
2. Each number is independent.

Random number is symbolized with U and these values are between 0 and 1, and can be stated $U(0,1)$. Various way to obtain random number can with computer random number table or use the random number method. One of method to generate random number is Linear Congruential Generator (LCG). LCG has formula as follows.

$$Z_i = (aZ_{i-1} + c) \bmod m$$

where Z_i is a series of random number n -th, Z_{i-1} is a previous random number, a is multiplier factor, c is a constant and m is modulo. The algorithm to generate standardized normal data with Box-Muller transformation as follows.

1. Generate U_1 and U_2 from $U(0,1)$.
2. Determine

$$Z_1 = (-2 \ln U_1)^{\frac{1}{2}} \cos(2\pi U_2), \text{ or}$$

$$Z_2 = (-2 \ln U_1)^{\frac{1}{2}} \sin(2\pi U_2)$$

3. So Z_1, Z_2 are random number from $U(0,1)$.
 - a. Generate Z from $N(0,1)$ in the previous algorithm.
 - b. Determine $X = \sigma Z + \mu$.

To generate number with Weibull distribution can be calculated with formula $X = \beta(-\ln U)^{\frac{1}{\alpha}}$ [5]. Probabilistic Economic Order Quantity (EOQ) model is a method that is used to determine when an order will be done and the quantity of goods in each order to minimize value of total cost. The following are steps to determine Q and R.

1. Calculate Q when $E(M > R) = 0$, so that Q can be calculated with formula:

$$Q = \sqrt{\frac{2AD}{H}}$$

where Q is quantity in unit, D is average of demand in year in unit and A is cost of order.

2. Determine P ($M > R$) and R using Q

$$P(M > R) = \frac{HQ}{KD}$$

with $R = \bar{M} + S$.

3. Determine E ($M > R$) using R

$$E(M > R) = \sum_{M=R+1}^{M_{max}} (M - R)P(M)$$

4. Redetermine Q with $E(M > R)$ as Q^*

$$Q^* = \sqrt{\frac{2D[A \times K \times E(M > R)]}{H}}$$

After obtaining Q^* , the next step performs iteration at step (2), (3) and (4) until Q and R are same.

Data that will be discussed is demand, cost, and lead time [6]. The calculation is performed with numerical calculation using probabilistic EOQ model and will be simulated using Monte Carlo simulation so that is obtained optimal solution of supply in the future.

3. Analysis and Discussion

The following table is data of demand of shoes.

Table 1 Data of Demand

No	Period	Demand
1	October, 2012	667
2	November, 2012	653
3	December, 2012	837
4	January, 2013	576
5	February, 2013	683
6	March, 2013	362
7	April, 2013	162
8	May, 2013	563
9	June, 2013	435
10	July, 2013	774
11	Agustus, 2013	476
12	September, 2013	381
13	October, 2013	707
14	November, 2013	847
15	December, 2013	559
16	January, 2014	578
17	February, 2014	896
18	March, 2014	838
19	April, 2014	773
20	May, 2014	742
21	June, 2014	644
22	July, 2014	745
23	Agustus, 2014	869
Total		14767
Average		642043

Meanwhile, data of cost that is needed to make shoes consist of price per-unit (P), cost of order (A), cost of saving (H) and cost of lack (K) per-year.

Table 2 Data of Cost

Product	Purchase Price (\$)	Purchase Order (\$)	Save Cost (\$)	Shortage Cost (\$)
Shoes	29,32	18,21	3,76	5

Table 3 Lead Time

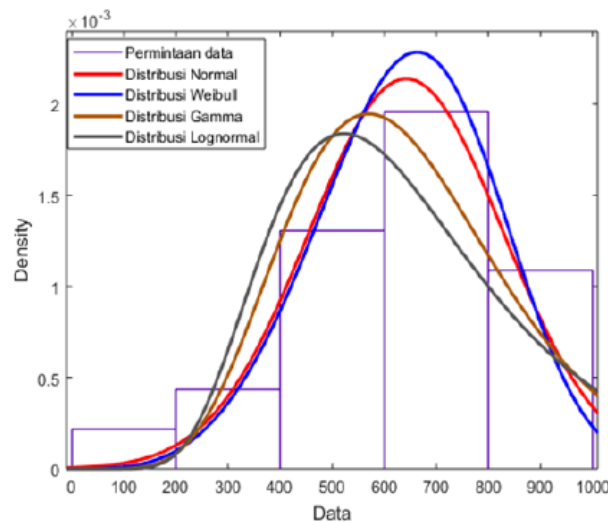
Product	per-Month	per-Year
Shoes	3	$\frac{1}{4}$

Based on calculation above, to obtain total of supply cost from demand of shoes can be determined with probabilistic EOQ supply model. After calculating the value of optimal order measure with several iteration, the following is a summary of calculation result Q, P ($M > R$), R and E ($M > R$).

Table 4 Calculation of Q, R, P (M > R) and E (M > R)

Iteration	Q (Unit)	P (M > R)	R (Unit)	E (M > R)
1	378	0.0192	2445	5.9047
2	613	0.0312	2406	9.619
3	722	0.0367	2388	11.333
4	767	0.039	2381	11.999
5	784	0.039	2381	11.999
6	784	0.039	2381	11.999

Based on Table 4 shows that the number of Q and R in several iteration have been done to obtain same value. The value of optimal demand (Q) is 784 unit and the return demand point (R) is 2381 unit. Using MATLAB R2017a is obtained curve of probability density function of demand as follow.

**Figure 1** Distribution Fitting PDF

Based on the Figure 1 the demand data more approach to Normal dan Weibull distribution rather than Gamma and Lognormal distribution. The value of log likelihood normal distribution and Weibull is -152,39 and -151,988, rather than log likelihood of Gamma and lognormal -155,304 and -157,83. So demand data more approach to Normal and Weibull distribution.

4. Conclusion

This research designs a simulation to control the inventory with number of demand that is probabilistic. The calculation is conducted with numerical calculation using probabilistic EOQ model and Monte Carlo simulation. Based on calculation, the inventory cost total using probabilistic EOQ model is \$486128,195. After simulation using Monte Carlo simulation where data of demand follows normal distribution and obtained the total of supply cost is \$461116,05 in 23 months, where data of demand using Weibull distribution obtained total of supply cost is \$482301,76. So that can be summarized that Monte Carlo simulation can calculate the supply of

optimal probabilistic in the future knowing the probability distribution following data of demand.

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